Semantics of communicating systems

A configuration $s' = \langle \vec{q'} \; ; \; \vec{b'} \rangle$ is *reachable* from another configuration $s = \langle \vec{q} \; ; \; \vec{b} \rangle$ by **firing a transition** l, written $s \xRightarrow{l} s'$, if either of the following holds

1. $l = \underline{\mathsf{A}}\,\mathsf{B}!\mathsf{m}$ and $q_{\mathsf{A}} \xrightarrow{l}_{\mathsf{A}} q'_{\mathsf{A}}$ and
   a. $q'_{\mathsf{C}} = q_{\mathsf{C}}$ for all $\mathsf{C} \neq \mathsf{A}$ and
   b. $b'_{\mathsf{AB}} = b_{\mathsf{AB}}.\mathsf{m}$ and
   c. $b'_{\mathsf{CD}} = b_{\mathsf{CD}}$ for all $(\mathsf{C}, \mathsf{D}) \neq (\mathsf{A}, \mathsf{B}) \in \mathcal{C}$

2. $l = \mathsf{A}\,\underline{\mathsf{B}}?\mathsf{m}$ and $q_{\mathsf{B}} \xrightarrow{l}_{\mathsf{B}} q'_{\mathsf{B}}$ and
   a. $q'_{\mathsf{C}} = q_{\mathsf{C}}$ for all $\mathsf{C} \neq \mathsf{B}$ and
   b. $b_{\mathsf{AB}} = \mathsf{m}.b'_{\mathsf{AB}}$ and
   c. $b'_{\mathsf{CD}} = b_{\mathsf{CD}}$ for all $(\mathsf{C}, \mathsf{D}) \neq (\mathsf{A}, \mathsf{B}) \in \mathcal{C}$

Special configurations

$s = \langle \vec{q} \; ; \; \vec{b} \rangle$ is **stable** if $\vec{b} = \vec{\varepsilon}$.
$s = \langle \vec{q} \; ; \; \vec{b} \rangle$ is a **deadlock** if $s \not\Rightarrow$ and there exists a participant $\mathsf{A} \in \mathcal{P}$ such that $q_{\mathsf{A}} \xrightarrow{\mathsf{A}\,\underline{\mathsf{B}}?\mathsf{m}}_{\mathsf{A}} q'_{\mathsf{A}}$

From global to local

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Projecting g-choreographies

- Functions $\_ \downarrow_{A}$ yield the projection of g-choreographies on the participant A

From global to local

Projecting g-choreographies

- Functions $\_ \downarrow_{\mathsf{A}}$ yield the projection of g-choreographies on the participant A as triplets (M, q_0, q_e) with q_0 and q_e initial and terminal states respectively
- Dis-jointly "combine" $\mathsf{G}_1 \downarrow_{\mathsf{A}}$ and $\mathsf{G}_2 \downarrow_{\mathsf{A}}$ with G_1 and G_2 sub-terms of G
- Let us define $(M, q_0, q_e) \otimes \ \mathbf{n}$ which transforms each state q of M in (q, n)

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Choice

$$(\mathsf{G}_1 + \mathsf{G}_2) \downarrow_{\mathsf{A}} = \left(\left\{ {}^{q_e^2}/_{q_e^1} \right\} M_1 \sqcup \left\{ {}^{q_0^1}/_{q_0^2} \right\} M_2, q_0^1, q_e^2 \right)$$

where $(M_1, q_0^1, q_e^1) = \mathsf{G}_1 \downarrow_{\mathsf{A}} \otimes \mathbf{1}$
and $(M_2, q_0^2, q_e^2) = \mathsf{G}_2 \downarrow_{\mathsf{A}} \otimes \mathbf{2}$

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Parallel composition

$$(\mathsf{G}_1 \mid \mathsf{G}_2) \downarrow_{\mathsf{A}} = \left(M_1 \times M_2, (q_0^1, q_0^2), (q_e^1, q_e^2)\right)$$

where $(M_1, q_0^1, q_e^1) = \mathsf{G}_1 \downarrow_{\mathsf{A}} \otimes \mathbf{1}$
and $(M_2, q_0^2, q_e^2) = \mathsf{G}_2 \downarrow_{\mathsf{A}} \otimes \mathbf{2}$

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Iteration

Let $\{\mathsf{A}, \mathsf{B}_1, \dots, \mathsf{B}_h\}$ be the set of participants of G

$$*\mathsf{G} \downarrow_{\mathsf{X}} = \left(M_1 \sqcup \left\{ {}^{q_e^1}/_{q_0^{\mathsf{G}}} \right\} M_{\mathsf{G}} \sqcup \left\{ {}^{q_e^{\mathsf{G}}}/_{q_0^2} \right\} M_2, q_0^1, q_e^2 \right) \qquad \text{where}$$

$$(M_{\mathsf{G}}, q_0^{\mathsf{G}}, q_e^{\mathsf{G}}) = \mathsf{G} \downarrow_{\mathsf{X}}$$

$$(M_1, q_0^1, q_e^1) = (\mathsf{A} \to \mathsf{B}_1 : \lambda_{\mathsf{q}_0} \mid \cdots \mid \mathsf{A} \to \mathsf{B}_{\mathsf{H}} : \lambda_{\mathsf{q}_0}) \downarrow_{\mathsf{X}} \qquad \lambda_{\mathsf{q}_0} \text{ fresh}$$

$$(M_2, q_0^2, q_e^2) = (\mathsf{A} \to \mathsf{B}_1 : \epsilon_{\mathsf{q}_e} \mid \cdots \mid \mathsf{A} \to \mathsf{B}_{\mathsf{H}} : \epsilon_{\mathsf{q}_e}) \downarrow_{\mathsf{X}} \qquad \epsilon_{\mathsf{q}_e} \text{ fresh}$$

A first result: Progress

Theorem If $[\![G]\!] \neq \perp$ then any reachable configuration from the initial configuration of $(\Delta(G \downarrow_A))_{A \in \mathcal{P}}$ is not a deadlock.

Proof sketch (structural induction on G).

Base cases: by construction.
Inductive step rely on determinisation of CFSMs since it preserves the language of the communicating system.
For sequential and parallel composition, we show that if there is a deadlock in the composed communicating system, then there must be a deadlock in at least one of the constituent
Choice: well-branchedness implies that traces of $(\Delta((G_1 + G_2) \downarrow_A))_{A \in \mathcal{P}}$ belong to either $(\Delta(G_1 \downarrow_A))_{A \in \mathcal{P}}$ or $(\Delta(G_2 \downarrow_A))_{A \in \mathcal{P}}$. We can build a simulation relation between the communicating system of the non-deterministic choice and the one consisting of the CFSMs $(G_i \downarrow_A)_{A \in \mathcal{P}}$.
Iteration: inductive hypothesis + well-formedness of additional interactions to decide the loop exit. □

Relating g-choreographies and communicating systems

G

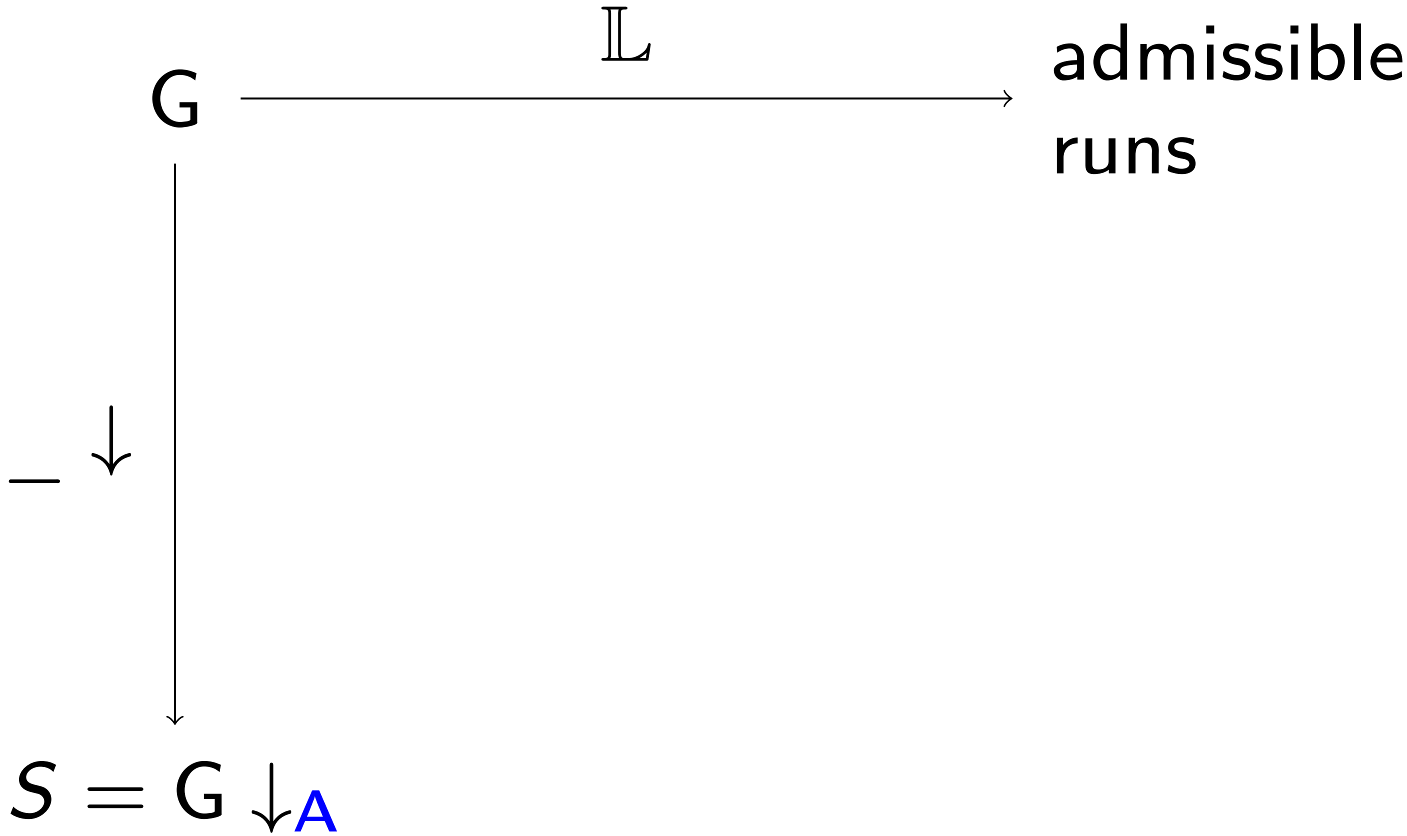
$$S = \mathsf{G} \downarrow_{\mathsf{A}}$$

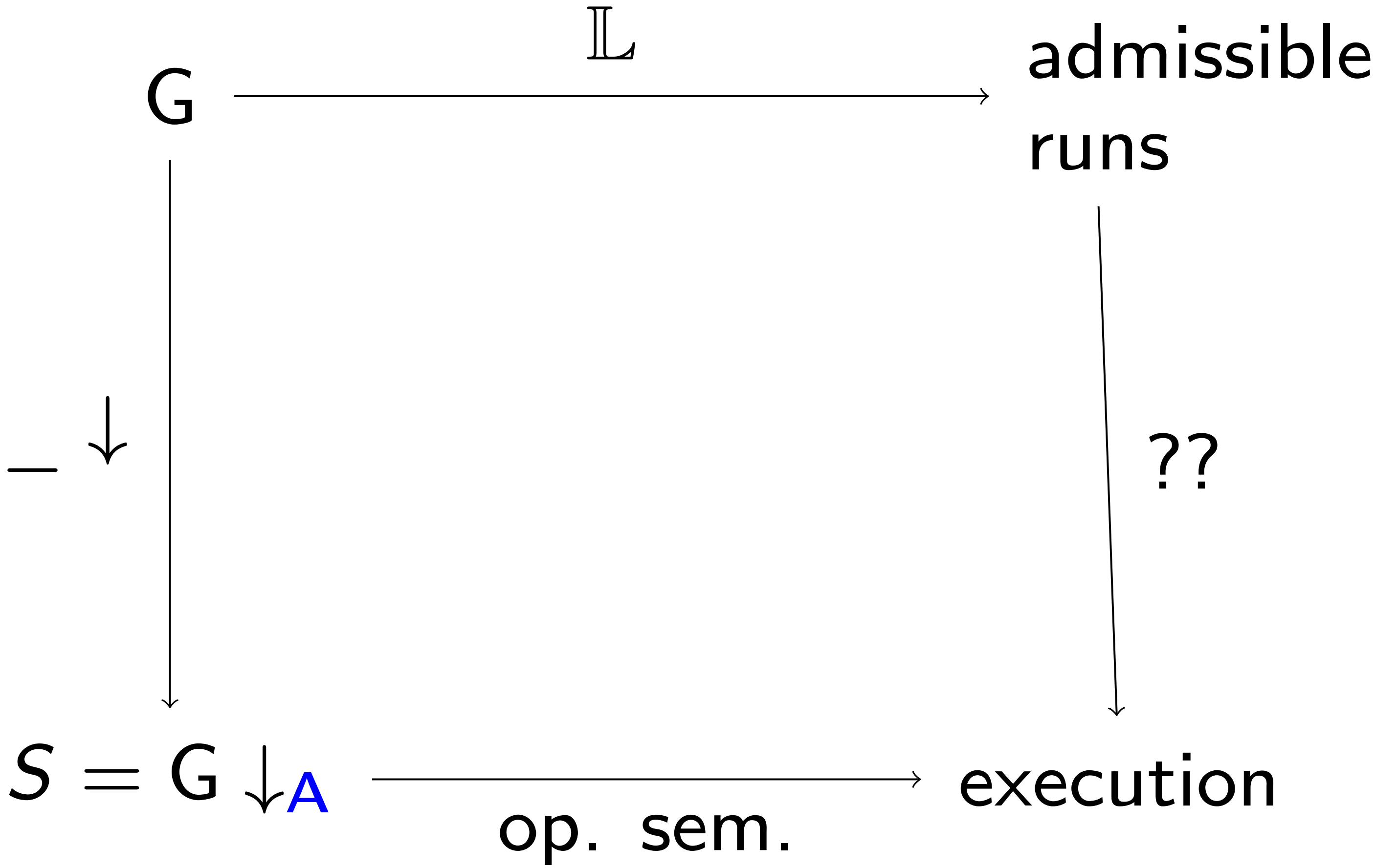
$$\mathsf{G}$$

$$\_\downarrow$$

$$S = \mathsf{G}\downarrow_{\mathsf{A}}$$

Relating g-choreographies and communicating systems




G
$\mathbb{L}$
admissible runs
$\_ \downarrow$
??
$S = G \downarrow_{A}$
op. sem.
execution

Language of g-choreographies

The language of a g-choreography with a defined semantics consists of words on the communication actions in $\mathcal{L}$ that occur in the g-choreography.

Language of g-choreographies

The language of a g-choreography with a defined semantics consists of words on the communication actions in $\mathcal{L}$ that occur in the g-choreography.
Let $r = [\mathcal{E}, \leq, \lambda]$, the **language** $\mathbb{L}[r]$ of r is the set of $\lambda(w)$ where $w \in \mathcal{E}^\star$ is s.t. for any $0 \leq i \neq j \leq \text{len}(w)$

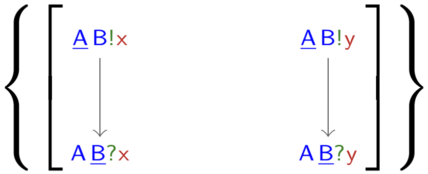
1. $w[i] \neq w[j]$
2. if $w[i] \leq w[j]$ then $i \leq j$
3. for every $e \in \mathcal{E}$, if there is i s.t. $e < w[i]$ then $e \in w$

The **language** of $\mathsf{G} \in \mathcal{G}$ is

$$\mathbb{L}[\mathsf{G}] = \begin{cases} \bigcup_{r \in [\![\mathsf{G}]\!]} \mathbb{L}[r], & \text{if } [\![\mathsf{G}]\!] \text{ is defined} \\ \text{undef}, & \text{otherwise} \end{cases}$$

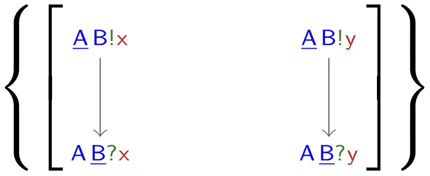
Examples of languages of g-choreography

$G_1 = A \rightarrow B : x \mid A \rightarrow B : y$

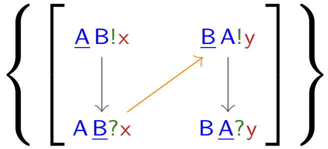


Examples of languages of g-choreography

$G_1 = A \to B : x \mid A \to B : y$



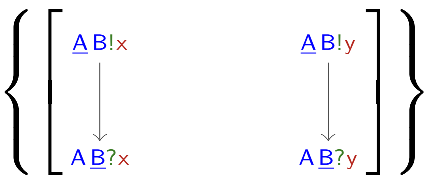
$G_2 = A \to B : x ; B \to A : y$



$$
\begin{aligned}
\mathbb{L}[G_1] = \{ & \underline{A}\,B!x\;\underline{A}\,B!y\;A\,\underline{B}?x\;A\,\underline{B}?y, \\
& \underline{A}\,B!y\;\underline{A}\,B!x\;A\,\underline{B}?x\;A\,\underline{B}?y, \\
& \underline{A}\,B!x\;\underline{A}\,B!y\;A\,\underline{B}?y\;A\,\underline{B}?x, \\
& \underline{A}\,B!y\;\underline{A}\,B!x\;A\,\underline{B}?y\;A\,\underline{B}?x, \\
& \underline{A}\,B!x\;A\,\underline{B}?x\;\underline{A}\,B!y\;A\,\underline{B}?y, \\
& \underline{A}\,B!y\;A\,\underline{B}?y\;\underline{A}\,B!x\;A\,\underline{B}?x, \\
& + \text{ all the prefixes}\}
\end{aligned}
$$

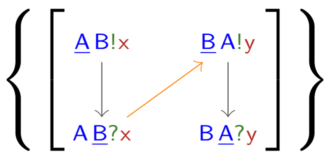
Examples of languages of g-choreography

$G_1 = A{\to}B: x \mid A{\to}B: y$



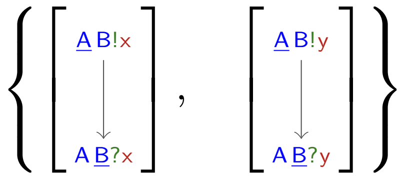
$$
\begin{aligned}
\mathbb{L}[G_1] = \{ & \underline{A}\,B!x\;\underline{A}\,B!y\;A\,\underline{B}?x\;A\,\underline{B}?y, \\
& \underline{A}\,B!y\;\underline{A}\,B!x\;A\,\underline{B}?x\;A\,\underline{B}?y, \\
& \underline{A}\,B!x\;\underline{A}\,B!y\;A\,\underline{B}?y\;A\,\underline{B}?x, \\
& \underline{A}\,B!y\;\underline{A}\,B!x\;A\,\underline{B}?y\;A\,\underline{B}?x, \\
& \underline{A}\,B!x\;A\,\underline{B}?x\;\underline{A}\,B!y\;A\,\underline{B}?y, \\
& \underline{A}\,B!y\;A\,\underline{B}?y\;\underline{A}\,B!x\;A\,\underline{B}?x, \\
& + \text{ all the prefixes}\}
\end{aligned}
$$

$G_2 = A{\to}B: x; B{\to}A: y$



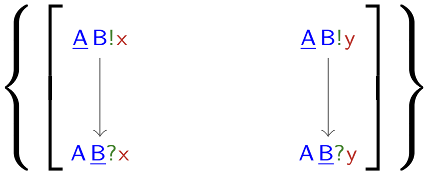
$$
\begin{aligned}
\mathbb{L}[G_2] = \{ & \underline{A}\,B!x\;A\,\underline{B}?x\;\underline{B}\,A!y\;B\,\underline{A}?y \\
& + \text{ all the prefixes}\}
\end{aligned}
$$

$G_3 = A{\to}B: x + A{\to}B: y$



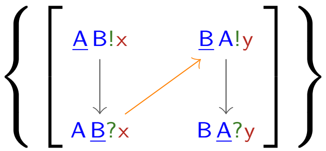
Examples of languages of g-choreography

$G_1 = A{\to}B: x \mid A{\to}B: y$



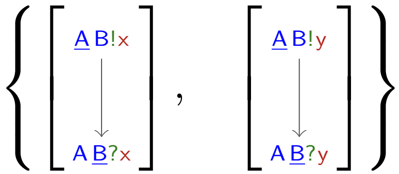
$$
\begin{aligned}
\mathbb{L}[G_1] = \{ & \underline{A}\,B!x\ \underline{A}\,B!y\ A\,\underline{B}?x\ A\,\underline{B}?y, \\
& \underline{A}\,B!y\ \underline{A}\,B!x\ A\,\underline{B}?x\ A\,\underline{B}?y, \\
& \underline{A}\,B!x\ \underline{A}\,B!y\ A\,\underline{B}?y\ A\,\underline{B}?x, \\
& \underline{A}\,B!y\ \underline{A}\,B!x\ A\,\underline{B}?y\ A\,\underline{B}?x, \\
& \underline{A}\,B!x\ A\,\underline{B}?x\ \underline{A}\,B!y\ A\,\underline{B}?y, \\
& \underline{A}\,B!y\ A\,\underline{B}?y\ \underline{A}\,B!x\ A\,\underline{B}?x, \\
& + \text{ all the prefixes}\}
\end{aligned}
$$

$G_2 = A{\to}B: x; B{\to}A: y$



$$
\begin{aligned}
\mathbb{L}[G_2] = \{ & \underline{A}\,B!x\ A\,\underline{B}?x\ \underline{B}\,A!y\ B\,\underline{A}?y \\
& + \text{ all the prefixes}\}
\end{aligned}
$$

$G_3 = A{\to}B: x + A{\to}B: y$



$$
\begin{aligned}
\mathbb{L}[G_3] = \{ & \underline{A}\,B!x\ A\,\underline{B}?x, \\
& \underline{A}\,B!y\ A\,\underline{B}?y, \\
& \underline{A}\,B!x, \\
& \underline{A}\,B!y \\
& \}
\end{aligned}
$$

Adequacy

Theorem If $\mathsf{G} \in \mathcal{G}$ with $[\![\mathsf{G}]\!] \neq \perp$ and $S = (\Delta(\mathsf{G} \downarrow_{\mathsf{A}}))_{\mathsf{A} \in \mathcal{P}}$ then $\mathbb{L}[S] \subseteq \mathbb{L}[\mathsf{G}]$.

Proof sketch (by structural induction on G).

Proof obligations

(*i*) dependencies are preserved by sequential composition

(*ii*) parallel composition does not yield unexpected communication.

(*i*): by definition, every word w_0 in $\mathbb{L}[\mathsf{G}; \mathsf{G}']$ is the order-preserving shuffling of two words, $w \in \mathbb{L}[\mathsf{G}]$ and $w' \in \mathbb{L}[\mathsf{G}']$

+

by well-sequencedness all events in w with subject A precede in w_0 every event of w' with subject A.

(*ii*): as for the progress theorem. □

The semantics are not equivalent

Projected communicating system can have less behaviours than their protocol due to the FIFO policy on buffers.

Example

Take the g-choreography

$$\mathsf{G} = \mathsf{A}{\rightarrow}\mathsf{B}\colon \mathsf{x} \mid \mathsf{A}{\rightarrow}\mathsf{B}\colon \mathsf{y}$$

we have

$$\underline{\mathsf{A}}\,\mathsf{B}!\mathsf{x}\;\underline{\mathsf{A}}\,\mathsf{B}!\mathsf{y}\;\mathsf{A}\,\underline{\mathsf{B}}?\mathsf{y}\;\mathsf{A}\,\underline{\mathsf{B}}?\mathsf{x} \in \mathbb{L}[\mathsf{G}] \setminus \mathbb{L}[(\Delta(\mathsf{G}\downarrow_{\mathsf{A}}))_{\mathsf{A}\in\mathcal{P}}]$$

References I

[1] Gul Agha. *Actors: A Model of Concurrent Computation in Distributed Systems.* MIT Press, Cambridge, MA, USA, 1986.

[2] Daniel Brand and Pitro Zafiropulo. On Communicating Finite-State Machines. *JACM*, 30(2):323–342, 1983.

[3] Pierre-Malo Deniélou and Nobuko Yoshida. Multiparty session types meet communicating automata. In *ESOP 2012*, pages 194–213, 2012.

[4] Michael J. Fischer, Nancy A. Lynch, and Michael S. Paterson. Impossibility of distributed consensus with one faulty process. *J. ACM*, 32(2):374–382, apr 1985.

[5] Roberto Guanciale and Emilio Tuosto. An abstract semantics of the global view of choreographies. In *Proceedings 9th Interaction and Concurrency Experience, ICE 2016, Heraklion, Greece, 8-9 June 2016.*, pages 67–82, 2016.

[6] Roberto Guanciale and Emilio Tuosto. Realisability of pomsets. *Journal of Logic and Algebraic Methods in Programming*, 108:69–89, 2019.

[7] Carl Hewitt, Peter Boehler Bishop, and Richard Steiger. A Universal Modular ACTOR Formalism for Artificial Intelligence. In Nils J. Nilsson, editor, *Proceedings of the 3rd International Joint Conference on Artificial Intelligence. Standford, CA, USA, August 20-23, 1973*, pages 235–245. William Kaufmann, 1973.

[8] Nickolas Kavantzas, Davide Burdett, Gregory Ritzinger, Tony Fletcher, and Yves Lafon. Web services choreography description language version 1.0. http://www.w3.org/TR/2004/WD-ws-cdl-10-20041217. Working Draft 17 December 2004.

References II

[9] Roland Kuhn, Hernán C. Melgratti, and Emilio Tuosto. Behavioural types for local-first software.
In Karim Ali and Guido Salvaneschi, editors, *37th European Conference on Object-Oriented Programming, ECOOP 2023, July 17-21, 2023, Seattle, Washington, United States*, volume 263 of *LIPIcs*, pages 15:1–15:28. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2023.

[10] Julien Lange, Emilio Tuosto, and Nobuko Yoshida. From Communicating Machines to Graphical Choreographies.
In *POPL15*, pages 221–232, 2015.

[11] Emilio Tuosto and Roberto Guanciale. Semantics of global view of choreographies.
Journal of Logic and Algebraic Methods in Programming, 95:17–40, 2018. Revised and extended version of [5]. available at `http://www.cs.le.ac.uk/people/et52/jlamp-with-proofs.pdf`.