Some bureaucracy before moving on

$X \uplus Y \triangleq X \otimes \mathbf{1} \cup Y \otimes \mathbf{2}$ where

- $\mathbf{n} \triangleq \{n\}$ for any natural number n
- $f \otimes \mathbf{n} \triangleq \{(x, n) \mapsto f(x) \mid x \in X\}$ for any f on X
- $R \otimes \mathbf{n} \triangleq \{((x, n), (y, n)) \mid (x, y) \in R\}$ for any relation $R \subseteq X \times Y$

Given a pomset $r = [\mathcal{E}, \leq, \lambda]$, let $\mathcal{E}_{r,\mathsf{A}} \subseteq \mathcal{E}$ denote the set of events labelled with actions performed by A.

The **pomset of A in** r is

$$r{\downharpoonleft}_{\mathsf{A}} \quad = \quad [\mathcal{E}_{r,\mathsf{A}},\ \leq \cap\ (\mathcal{E}_{r,\mathsf{A}} \times \mathcal{E}_{r,\mathsf{A}}),\ \lambda|_{\mathcal{E}_{r,\mathsf{A}}}]$$

Well-sequencedness, formally

Let $r = [\mathcal{E}, \leq, \lambda]$ and $r' = [\mathcal{E}', \leq', \lambda']$ and define

$$\begin{aligned} \mathsf{seq}(r, r') &= [\mathcal{E} \uplus \mathcal{E}', \leq_{seq}, (\lambda \otimes \mathbf{1}) \cup (\lambda' \otimes \mathbf{2})] \\ \leq_{seq} &= \left((\leq \otimes \mathbf{1}) \cup (\leq' \otimes \mathbf{2}) \cup \bigcup_{\mathsf{A} \in \mathcal{P}} ((\mathcal{E}_{r,\mathsf{A}} \times \mathbf{1}) \times (\mathcal{E}_{r',\mathsf{A}} \times \mathbf{2})) \right)^{\star} \end{aligned}$$

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Two g-choreographies $\mathsf{G}, \mathsf{G}' \in \mathcal{G}$ are **well-sequenced** if

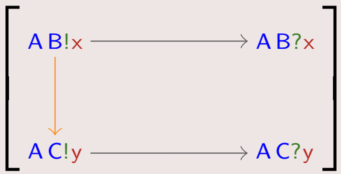
- $[\![\mathsf{G}]\!] \neq \mathsf{undef}$
- $[\![\mathsf{G}']\!] \neq \mathsf{undef}$, and
- $ws(r, r')$ for all $r \in [\![\mathsf{G}]\!]$, $r' \in [\![\mathsf{G}']\!]$

where $ws(r, r')$ if

- for all $e' \in \mathcal{E}'$ s.t. $\lambda'(e') = \mathsf{A\,B!m} \in \mathcal{E}$, if there is $e \in \mathcal{E}$ s.t. $\lambda(e) = \mathsf{A\,B?m}$ then $e \leq_{seq} e'$
- and $\leq_{\mathsf{seq}(r,r')} \supseteq (\{e \in \mathcal{E} \mid \lambda(e) \in \mathcal{L}^{!}\} \times \mathbf{1}) \times (\{e \in \mathcal{E}' \mid \lambda'(e) \in \mathcal{L}^{?}\} \times \mathbf{2})$

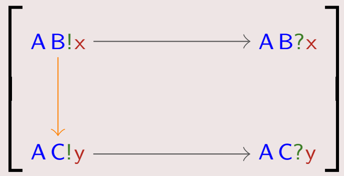
Well-sequencedness at work

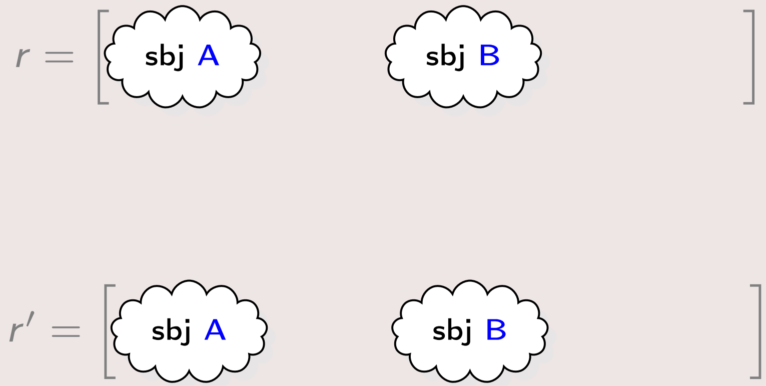
A→B : x; A→C : y



Well-sequencedness at work

$A \to B : x; A \to C : y$





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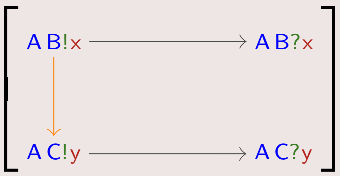
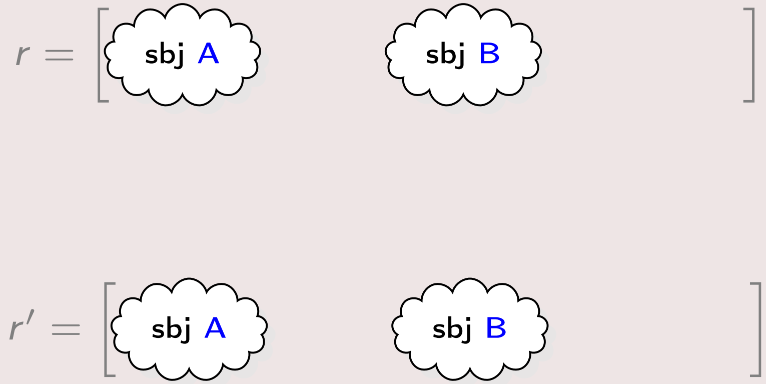
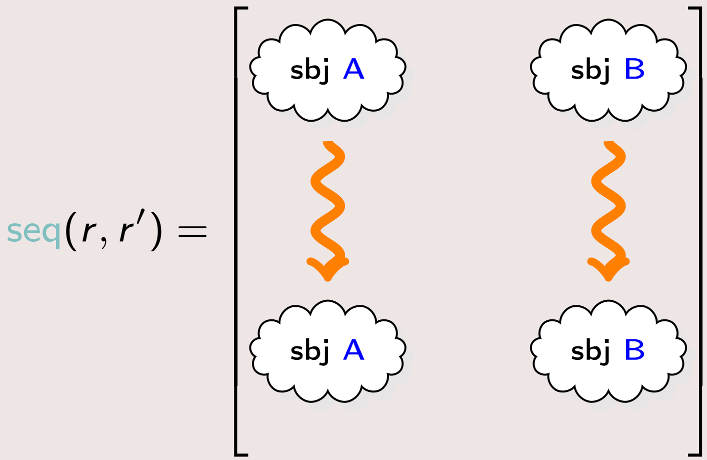
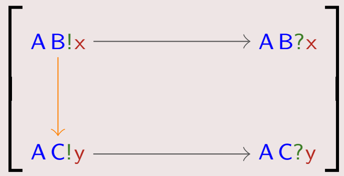






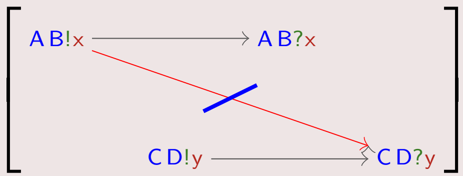
Well-sequencedness at work

A→B: x; A→C: y

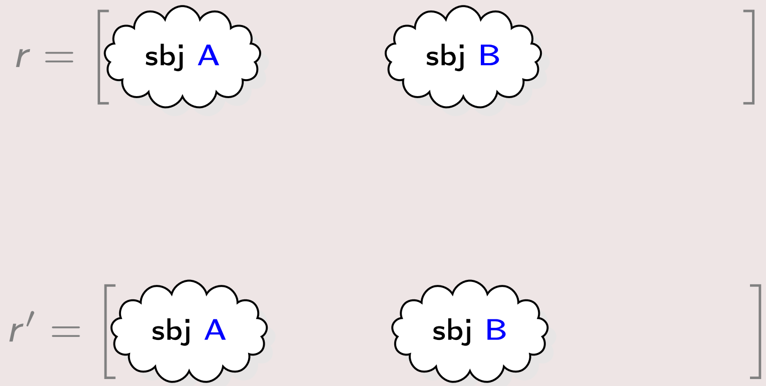


A→B: x; C→D: y

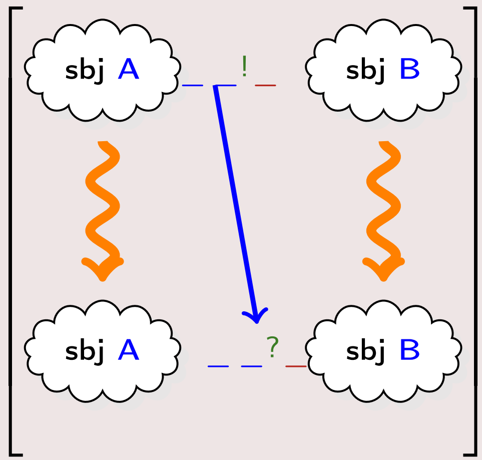
No minimal input "from the continuation"



(and no output in the continuation interfering with "left-inputs")



$ws(r, r')$



Well-branchedness: an intuition

In a branch $G_1 + G_2$

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- there should be at most one active participant non-standard

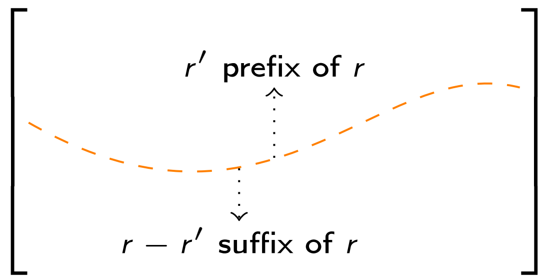
Well-branchedness: an intuition

In a branch $G_1 + G_2$

- there should be at most one active participant — non-standard
- any non-active participant should be passive — standard

Problem: Participants do not necessarily "enter" a choice "immediately"
An idea: find a "common part" of the branches for which participants behave uniformly in G_1 and G_2

$r =$



Prefix maps & divergence points

The pair of functions (ϕ, ψ) is an A-**prefix map of** G_1 **and** G_2 if

- $\operatorname{dom}\phi = \operatorname{dom}\psi$ is a partition of $[\![\mathsf{G}_1]\!]\lfloor_{\mathsf{A}}$ and $\operatorname{cod}\phi$ is a partition of $[\![\mathsf{G}_2]\!]\lfloor_{\mathsf{A}}$
- ϕ is bijective and $\operatorname{cod}\psi$ is a set of pomsets

s.t. for all $R \in \operatorname{dom}\phi$ and $(r, r') \in R \times \phi(R)$, the pomset $\psi(R)$ is a prefix r and r'.

Prefix maps & divergence points

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s.t. for all $R \in \mathsf{dom}\,\phi$ and $(r, r') \in R \times \phi(R)$, the pomset $\psi(R)$ is a prefix r and r'.

The **divergence point of G_1 and $\mathsf{G}_2$with respect to the A-prefix map (ϕ, ψ)** is

$$\mathsf{div}_{\phi,\psi}(\mathsf{G}_1, \mathsf{G}_2) = (L_1, L_2) \quad \text{where} \quad \begin{cases} L_1 = & \bigcup_{R \in \mathsf{dom}\,\phi} \bigcup_{r \in R} \lambda_r\big(\min\big(r - \psi(R)\big)\big) \\ L_2 = & \bigcup_{R \in \mathsf{cod}\,\phi} \bigcup_{r \in R} \lambda_r\big(\min\big(r - \psi(\phi^{-1}(R))\big)\big) \end{cases}$$

Active & passive components

Active & passive components

- A is **active** in $G_1 + G_2$ if there exists an A-prefix map (ϕ, ψ) of G_1 and G_2 with divergence point $\mathrm{div}_{\phi,\psi}(G_1, G_2) = (L_1, L_2)$ s.t.
  - $L_1 \cup L_2 \subseteq \mathcal{L}^!$
  - $L_1 \neq \emptyset$ and $L_2 \neq \emptyset$
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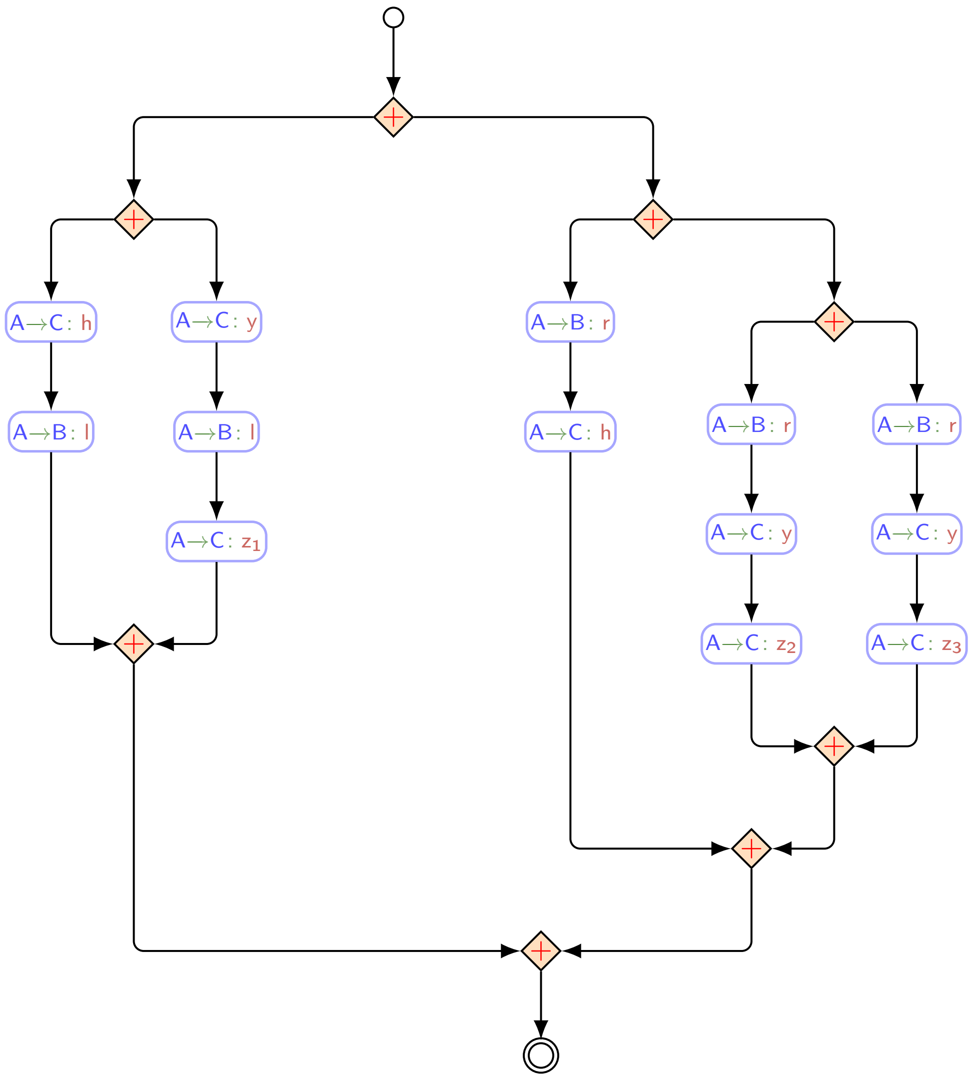
  - $L_1 \cap L_2 = \emptyset$
- A is **passive** in $G_1 + G_2$ if there exists an A-prefix map (ϕ, ψ) of G_1 and G_2 with divergence point $\mathsf{div}_{\phi,\psi}(G_1, G_2) = (L_1, L_2)$ s.t.
  - $L_1 \cup L_2 \subseteq \mathcal{L}^?$
  - $L_1 = \emptyset \iff L_2 = \emptyset$
  - $L_2 \cap \lambda_r(\mathcal{E}_{r-\psi(R)}) = \emptyset$ for all $R \in \operatorname{dom} \phi$ and $r \in R$
  - $L_1 \cap \lambda_r(\mathcal{E}_{r-\psi(\phi^{-1}(R))}) = \emptyset$ for all $R \in \operatorname{cod} \phi$ and $r \in R$

Class test

Figure out the graphical structure of the following terms and for each of them say which one is well-branched

- $G_1 = A \to B : \text{int} + A \to B : \text{str}$
- $G_2 = A \to B : \text{int} + \circledcirc$
- $G_3 = A \to B : \text{int} + A \to C : \text{str}$
- $G_4 = \left(\begin{array}{l} A \to C : \text{int}; A \to B : \text{bool} \\ \quad + \\ A \to C : \text{str}; A \to C : \text{bool}; A \to B : \text{bool} \end{array} \right)$

G😕: a difficult choice



G😕 = $G_1 + G_2$ where

$$G_1 = \left(\begin{array}{l} A \to C: h;\ A \to B: l \\ \quad + \\ A \to C: y;\ A \to B: l;\ A \to C: z_1 \end{array} \right)$$

$$G_2 = A \to B: r;\ A \to C: h + G_{2a} + G_{2b}$$

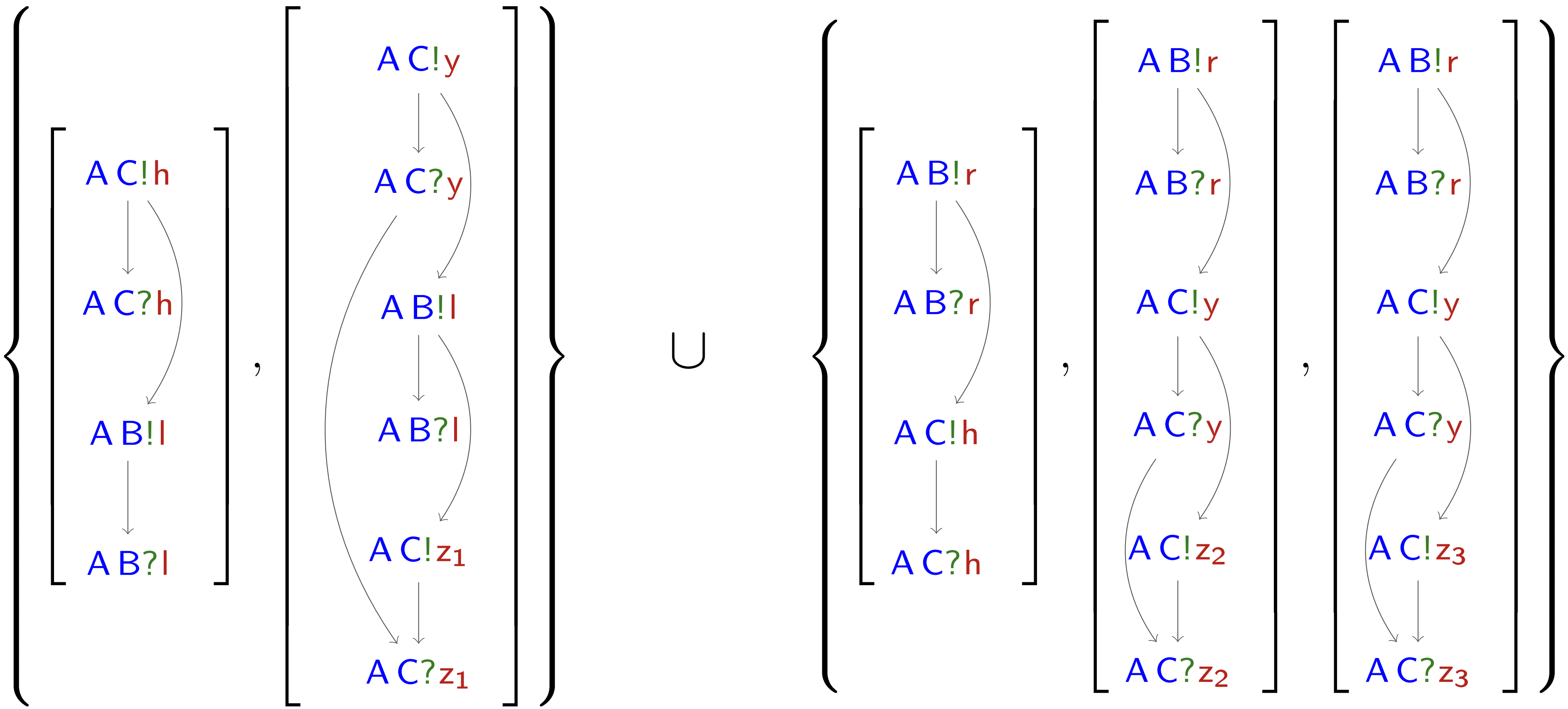
$$G_{2a} = A \to B: r;\ A \to C: y;\ A \to C: z_2$$

$$G_{2b} = A \to B: r;\ A \to C: y;\ A \to C: z_2$$

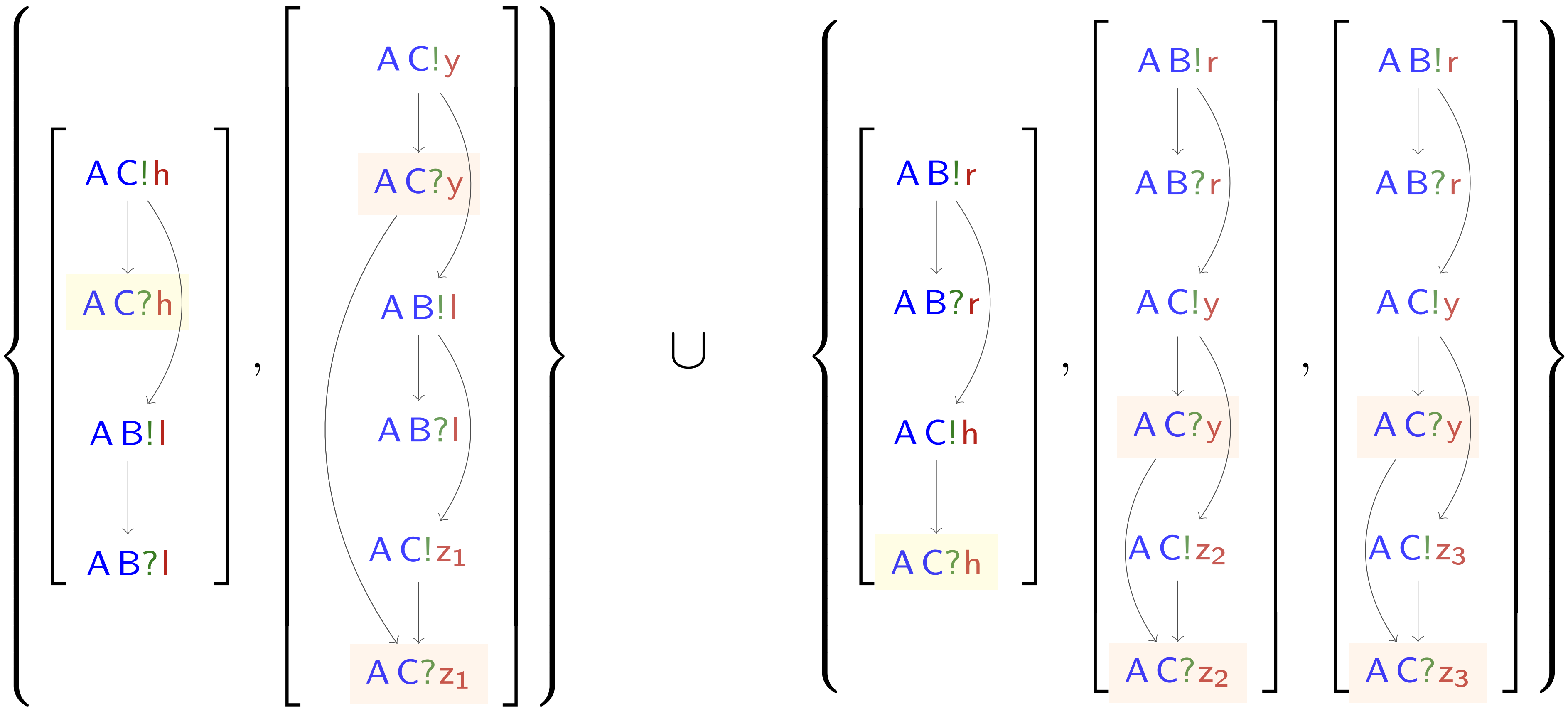
A chooses ... and

- Whatever B gets, he won't know if A and C exchanged or not h
- If C gets h, he won't know if A and B exchanged l or r

The pomsets of G🙁



The pomsets of G…from C's point of view



Communicating systems [2]

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A tuple $M = (Q, q_0, \rightarrow)$ is a **communicating finite-state machine** (CFSM) if

- Q is a finite set of *states* with $q_0 \in Q$ the **initial** state, and
- $\rightarrow \subseteq Q \times \mathcal{L} \times Q$; we write $q \xrightarrow{l} q'$ for $(q, l, q') \in \rightarrow$

M is **A-local** if for each $q \xrightarrow{l} q'$ $\begin{cases} \text{either} & l = \mathsf{B\,A?m} \\ \text{or} & l = \mathsf{A\,B!m} \end{cases}$ for some $\mathsf{B} \in \mathcal{P}$ and $\mathsf{m} \in \mathcal{M}$

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A **configuration** of S is a pair $s = \langle \vec{q}\ ;\ \vec{b} \rangle$ where

- $\vec{q} = (q_{\mathsf{A}})_{\mathsf{A} \in \mathcal{P}}$ with q_{A} a state of $S(\mathsf{A})$
- $\vec{b} = (b_{\mathsf{A\,B}})_{\mathsf{A\,B} \in \mathcal{C}}$ with $b_{\mathsf{A\,B}} \in \mathcal{M}^*$

The **initial** configuration s_0 is the one where, for all $\mathsf{A} \in \mathcal{P}$, q_{A} is the initial state of $S(\mathsf{A})$ and $b_{\mathsf{A\,B}} = \varepsilon$ for all $\mathsf{A\,B} \in \mathcal{C}$

Semantics of communicating systems

A configuration $s' = \langle \vec{q'} \;;\; \vec{b'} \rangle$ is *reachable* from another configuration $s = \langle \vec{q} \;;\; \vec{b} \rangle$ by **firing a transition** l, written $s \stackrel{l}{\Rightarrow} s'$, if either of the following holds

1. $l = \mathsf{A\,B!m}$ and $q_{\mathsf{A}} \xrightarrow{l}_{\mathsf{A}} q'_{\mathsf{A}}$ and
   a. $q'_{\mathsf{C}} = q_{\mathsf{C}}$ for all $\mathsf{C} \neq \mathsf{A}$ and
   b. $b'_{\mathsf{AB}} = b_{\mathsf{AB}}.\mathsf{m}$ and
   c. $b'_{\mathsf{CD}} = b_{\mathsf{CD}}$ for all $(\mathsf{C}, \mathsf{D}) \neq (\mathsf{A}, \mathsf{B}) \in \mathcal{C}$

2. $l = \mathsf{A\,B?m}$ and $q_{\mathsf{B}} \xrightarrow{l}_{\mathsf{B}} q'_{\mathsf{B}}$ and
   a. $q'_{\mathsf{C}} = q_{\mathsf{C}}$ for all $\mathsf{C} \neq \mathsf{B}$ and
   b. $b_{\mathsf{AB}} = \mathsf{m}.b'_{\mathsf{AB}}$ and
   c. $b'_{\mathsf{CD}} = b_{\mathsf{CD}}$ for all $(\mathsf{C}, \mathsf{D}) \neq (\mathsf{A}, \mathsf{B}) \in \mathcal{C}$

Special configurations

$s = \langle \vec{q} \;;\; \vec{b} \rangle$ is **stable** if $\vec{b} = \vec{\varepsilon}$.
$s = \langle \vec{q} \;;\; \vec{b} \rangle$ is a **deadlock** if $s \not\Rightarrow$ and there exists a participant $\mathsf{A} \in \mathcal{P}$ such that $q_{\mathsf{A}} \xrightarrow{\mathsf{A\,B?m}}_{\mathsf{A}} q'_{\mathsf{A}}$

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