Session Types, Exercise Sheet

1. Give a full type derivation of the following expressions, using an appropriate typing context. You also need to define the missing type rules by yourself.

   $x < y \vee x \oplus y = 10 \wedge y < 3$

2. Are the following pairs of types dual? Justify your answer. We assume $\mathsf{Alice}\dagger = \mathsf{Bob}$ and $\mathsf{Bob}\dagger = \mathsf{Alice}$

   (a) $S_1 = \mathsf{Alice}![\mathtt{bool}]; \mu \mathbf{t}.\mathsf{Alice}?[\mathtt{string}]; \mathsf{Alice}![\mathtt{bool}]; \mathbf{t}$ and
   $S_2 = \mu \mathbf{t}.\mathsf{Bob}?[\mathtt{bool}]; \mathsf{Bob}![\mathtt{string}]; \mathsf{Bob}?[\mathtt{bool}]; \mathbf{t}$

   (b) $S_1 = \mathsf{Alice}![\mathtt{int}]; \mu \mathbf{t}.\mathsf{Alice}?[\mathtt{bool}]; \mathsf{Alice} \oplus \left\{ \begin{array}{l} \mathit{continue} : \mathsf{Alice}![\mathtt{int}]; \mathbf{t} \\ \mathit{end} : \mathsf{Alice}![\mathtt{bool}]; \mathtt{end} \end{array} \right\}$
   and
   $S_2 = \mathsf{Bob}?[\mathtt{int}]; \mathsf{Bob}![\mathtt{bool}]; \mu \mathbf{t}.\mathsf{Bob} \& \left\{ \begin{array}{l} \mathit{continue} : \mathsf{Bob}?[\mathtt{int}]; \mathsf{Bob}![\mathtt{bool}]; \mathbf{t} \\ \mathit{end} : \mathsf{Bob}?[\mathtt{bool}]; \mathtt{end} \end{array} \right\}$

3. Derive subtyping for the following pairs (Derive $S_1 \leqslant S_2$):

   (a) $S_1 = \mathsf{Bob}![\mathtt{string}]; \mathsf{Bob}?[\mathtt{int}]; \mathtt{end}$ and $S_2 = \mathsf{Bob}![\mathtt{string}]; \mathsf{Bob}?[\mathtt{int}]; \mathtt{end}$

   (b) $S_1 = \mathsf{Bob}![\mathtt{string}]; \mathsf{Bob} \oplus \left\{ \begin{array}{l} \mathit{buy} : \mathsf{Bob} \& \left\{ \begin{array}{l} \mathit{red} : \mathsf{Bob}?[\mathtt{int}]; \mathsf{Bob}![\mathtt{bool}]; \mathtt{end} \\ \mathit{green} : \mathsf{Bob}?[\mathtt{bool}]; \mathtt{end} \\ \mathit{blue} : \mathsf{Bob}![\mathtt{int}]; \mathsf{Bob}![\mathtt{string}]; \mathtt{end} \\ \mathit{yellow} : \mathtt{end} \end{array} \right\} \\ \mathit{cancel} : \mathsf{Bob}![\mathtt{bool}]; \mathtt{end} \end{array} \right\}$
   and

$$S_2 = \mathsf{Bob}!\texttt{[string]}; \mathsf{Bob}\oplus\left\{\begin{array}{l} \mathit{buy} : \mathsf{Bob}\&\left\{\begin{array}{l}\mathit{red} : \mathsf{Bob}?\texttt{[int]}; \mathsf{Bob}!\texttt{[bool]}; \texttt{end} \\ \mathit{green} : \mathsf{Bob}?\texttt{[bool]}; \texttt{end} \\ \mathit{blue} : \mathsf{Bob}!\texttt{[int]}; \mathsf{Bob}!\texttt{[string]}; \texttt{end}\end{array}\right\} \\ \mathit{sell} : \mathsf{Bob}\&\left\{\begin{array}{l}\mathit{car} : \mathsf{Bob}?\texttt{[int]}; \mathsf{Bob}!\texttt{[bool]}; \texttt{end} \\ \mathit{bike} : \mathsf{Bob}?\texttt{[bool]}; \texttt{end}\end{array}\right\} \\ \mathit{cancel} : \mathsf{Bob}!\texttt{[bool]}; \texttt{end}\end{array}\right\}$$

(c) $S_1 = \mu \mathbf{t}.\mathsf{Alice}?\texttt{[int]}; \mathsf{Alice}\&\left\{\begin{array}{l}\mathit{continue} : \mathbf{t} \\ \mathit{terminate} : \mathsf{Alice}!\texttt{[bool]}; \texttt{end} \\ \mathit{cancel} : \texttt{end}\end{array}\right\}$ and

$S_2 = \mathsf{Alice}?\texttt{[int]}; \mu \mathbf{t}.\mathsf{Alice}\&\left\{\begin{array}{l}\mathit{continue} : \mathsf{Alice}?\texttt{[int]}; \mathbf{t} \\ \mathit{cancel} : \texttt{end}\end{array}\right\}$

4. Give a full type derivation of the following processes, with an appropriate session type:

(a) $P_{\mathsf{Bob}} = \overline{\mathsf{Alice}}\,\langle 25\rangle.\mathsf{Alice} \triangleright \left\{\begin{array}{l}\mathit{sword} : \overline{\mathsf{Alice}}\,\langle\text{“}\texttt{cyan}\text{”}\rangle.\mathbf{0} \\ \mathit{shield} : \overline{\mathsf{Alice}}\,\langle\text{“}\texttt{magenta}\text{”}\rangle.\mathbf{0} \\ \mathit{cancel} : \mathbf{0}\end{array}\right\}$

(b) $P_{\mathsf{Alice}} = \mathsf{Bob}\,(x).\texttt{if}\ x = 77$
$\texttt{then}\ \mathsf{Bob} \triangleleft \mathit{shield}.\mathsf{Bob}\,(y).\mathbf{0}$
$\texttt{else if}\ x = 83$
$\texttt{then}\ \mathsf{Bob} \triangleleft \mathit{sword}.\mathsf{Bob}\,(y).\mathbf{0}$
$\texttt{else}\ \mathsf{Bob} \triangleleft \mathit{cancel}.\mathbf{0}$

(c) $\mathsf{Alice} :: P_{\mathsf{Alice}} \mid \mathsf{Bob} :: P_{\mathsf{Bob}}$, with P_{Bob} and P_{Alice} the processes defined in (4a) and (4b) respectively.

5. Give a full type derivation of the following recursive processes, with an appropriate recursive session type:

$\mathsf{Bob}\,(x).\mu X.\mathsf{Bob} \triangleleft \mathit{shield}.\overline{\mathsf{Bob}}\,\langle x \oplus 0\rangle.X$