Modelling and Validation of Concurrent System: the calculus of communicating systems (CCS)

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Motivation

The "classical" models, according to Chomsky hierarchy

Operational / Denotational Models

1. Finite Automata / Regular Languages
   Represent finite-state systems
2. Push-Down Automata / Context-Free Languages
   Represent finite-state systems with a memory stack
3. Linear-Bounded Automata / Context-Sensitive Languages
   Represent finite-state systems with a finitely-long list as store
4. Turing Machines / Unrestricted Languages
   Represent finite-state systems with a infinitely-long list as store

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Isn’t this enough? Turing Machines are universal

Implement any computable function

One model per expressiveness class is enough, right?

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Programming Languages

- Low / High level
- General purpose / DSLs
- Imperative / Functional / Logic
- Object-Oriented / Aspect-Oriented / Service-Oriented

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The intended system matters!

What aspects consider when modelling concurrent systems?

(Non-)Termination

- Sequential programs implement "functionalities"
  - One expects them to terminate and (sometimes) return a result
  - Examples: factorial, bank account, queue

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- Sequential programs implement “functionalities”
  - One expects them to terminate and (sometimes) return a result
  - Examples: factorial, bank account, queue
- Concurrent programs implement “behaviour”
  - One expects them to (often) run forever, being reactive and responsive
  - Examples: operating system, cloud storage, social network

What aspects consider when modelling concurrent systems?

Reactiveness is key

Sequential programs implement "functionalities"

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- Examples: factorial, bank account, queue

Concurrent programs implement "behaviour"

- Are often idle (or with invisible activity), reacting to stimula
- Examples: ATM machine, sensor network, alarm system

How to consider...

Non-Termination

- Key aspect: accept infinite words
- (Simplest) Operational / Denotational Model
  Büchi Automata / ω-Regular Languages

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Interaction

- Key aspects: communication and parallelism
- (Simplest) Operational / Denotational Model
  Calculus of Communicating Systems (CCS) / Labelled Transition Systems

Consider a vending machine

(a typical non-terminating reactive machine):

- Intended (user) behaviour:
  insert a coin; choose coffee or tea; pick the beverage

Modelling and reasoning about reactive systems

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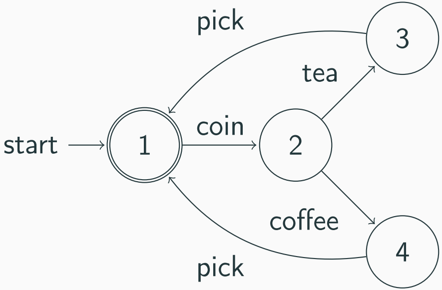

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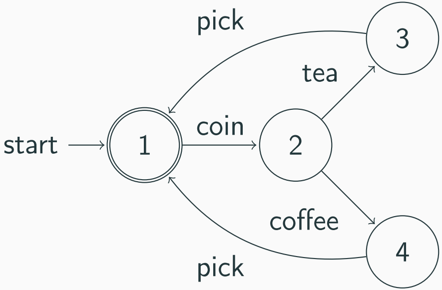

- Denotational model: $coin.(\mathit{coffee} + \mathit{tea}).\mathit{pick}$

Modelling and reasoning about reactive systems

- A regular expression specifies the intended system behaviour
- An automaton implements the intended system behaviour

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System Correctness: back to basics

The program (automaton) is *correct* if it implements (accepts) exactly the intended behaviour (the language of the regular expression).

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Is the vending machine correct?

- Language of the vending machine, by converting the automaton

$$coin.(coffee.pick + tea.pick)$$

- $coin.(coffee + tea).pick = coin.(coffee.pick + tea.pick)$

Is the vending machine correct?

- In the previous example we used the Kleene algebra distribution law (of sequencing over choice) to conclude the correctness of the automaton
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The vending machine is correct

Is the equivalence notion the right one for reactive systems?

An equivalent vending machine

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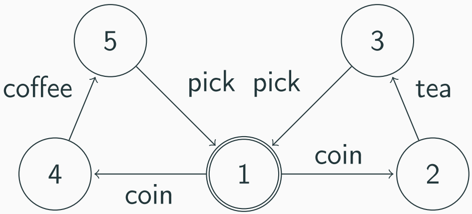

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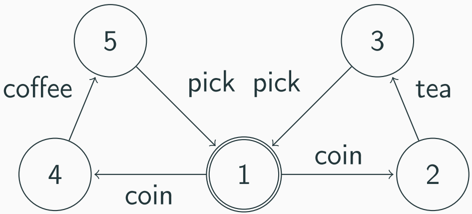

Does it have the same intended behaviour?

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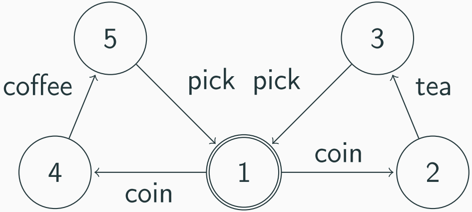

Does it have the same intended behaviour?

- When the user inserts the coin, the automaton non-deterministically decides to go to the left or to the right
- The user no longer can choose between tea or coffee...

Are automata adequate to reactive systems?

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  It should never terminate

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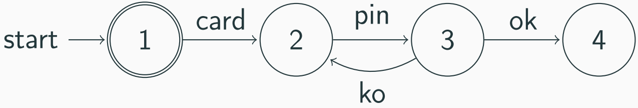

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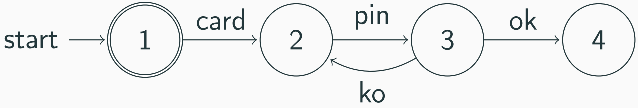

card and pin are user's actions; ok and ko are ATM's actions

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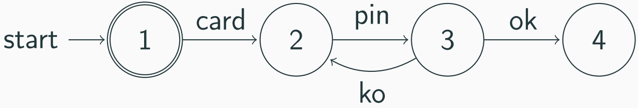

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  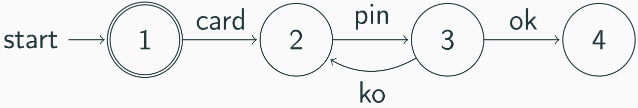

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  The interacting parties need to communicate synchronously,
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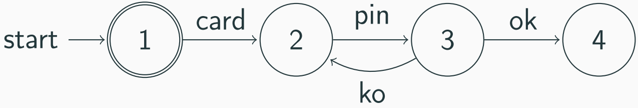

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- Imagine a synchronous like VOIP
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  An automaton does not represent synchronous communication

Requirements for an adequate model of reactive systems

In short

- Represent non-terminating behaviour
  without considering necessarily final states
- Distinguish input and output actions
  and allow the input of one party to be the output of another
- Support parallelism and communication
  systems composed by (a)synchronous interactive components
  interacting with their environment
- Use a finer notion of equivalence
  taking choice into consideration

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What is an appropriate operational / denotational model?

Calculus of Communicating Systems (CCS)

CCS: a process algebra

- Syntax: a language defined by a regular grammar
- Operational semantics: a transition relation
- Denotational semantics: a mathematical representation of non-terminating reactive systems

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Mathematical interpretation

Labelled Transition Systems, equipped with a congruence relation

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Mathematical interpretation

Labelled Transition Systems, equipped with a congruence relation

Congruence

Substitutive equivalence preserved by the operations of the language

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Why *calculus*?

A minimal mathematical language for calculations and reasoning

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Ockham's razor Principle: *lex parsimoniae*

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A minimal mathematical language for calculations and reasoning

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- Leibniz's infinitesimal calculus
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Why *minimal*?

Ockham's razor Principle: *lex parsimoniae*

Shaves off unnecessary hair – the best definition/explanation is the simplest one

Basically, what is a reactive system?

A process able of performing (interactive) actions and after each one, becoming another process

Motto (Tony Hoare and Robin Milner)

(In reactive systems) Everything is a process!

Remember set theory? In Mathematics, everything is a set

Calculus of Communicating Systems (CCS)

Assume a countable set $\mathcal{N}$ of action *names*; then CCS actions are defined as follows:

$\alpha ::=$		Actions
	a	Input action
	$\mid \overline{a}$	Output action
	$\mid \tau$	Silent (internal) action

a and $\overline{a}$ are observable actions, while τ is an unobservable action.

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Processes

A computing agent able of performing internal computation and of interacting with its environment via communicating actions.

Syntax of CCS

Processes

Consider for each process variable A a defining equation $A(x_1, \ldots, x_n) = P$ where the name variables $x_1, \ldots, x_n$ occur (bound) in P.

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$$
\begin{array}{llr}
P, Q, R ::= & & \text{Processes} \\
& 0 & \text{Empty process} \\
& \mid A\langle a_1, \ldots, a_n \rangle & \text{process definition} \\
& \mid \alpha.P & \text{action prefix} \\
& \mid (\mathbf{new}\, a)P & \text{action hiding} \\
& \mid P \mid Q & \text{parallel composition} \\
& \mid P + Q & \text{(non-deterministic) choice}
\end{array}
$$

Ingredients of CCS

- Actions are
  - *offers* (inputs)
  - *selections* (outputs),
  - or *idle* (invisible)
- Hiding makes actions invisible
- Parallel composition allows synchronous (by handshake) communication between two processes
- Definitions support generic processes and recursion

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Precedence in decreasing order

- hiding
- prefixing
- parallel composition
- choice

Vending Machine

In a process equation $A(x_1, \ldots, x_n)$ or definition $A\langle a_1, \ldots, a_n \rangle$, if $n = 0$ we simply write A

Version 1

- $VM = coin.(tea.\overline{pick}.VM + coffee.\overline{pick}.VM)$
- $Client = \overline{coin}.\overline{coffee}.pick.0$
- $System = VM \mid Client$

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Version 2

- $VM = coin.\overline{sugar}.(yes.fill.Serve + no.Serve)$
- Serve $= (tea.\overline{pick}.VM + coffee.\overline{pick}.VM)$

ATM

- $Box = card.Session$
- $Session = (pin.(\overline{ok}.Serve + \overline{ko}.Session) + exit.\overline{card}.Box)$
- $Serve = (balance.\overline{pick}.Session + deposit.amount.\overline{pick}.Session + withdraw.amount.\overline{pick}.Session)$
- $Client = \overline{card}.\overline{pin}.Use$
- $Use = (ok.\overline{balance}.pick.\overline{pin}.ok.\overline{withdraw}.\overline{Fifty}.pick.\overline{exit}.card.0 + ko.\overline{exit}.card.0)$
- $System = Box \mid Client$

Actions, free and bound actions, and action substitution

Actions of a process $\mathsf{Act}(P) \subseteq \mathsf{Act}$

is a set inductively defined by the following rules.

$$\begin{aligned}
\mathsf{Act}(A\langle a_1, \ldots, a_n \rangle) &= \{a_1, \ldots, a_n\} \\
\mathsf{Act}(\alpha.P) &= \{a\} \cup \mathsf{Act}(P), \ \text{if } \alpha = a \text{ or } \alpha = \overline{a} \\
\mathsf{Act}((\mathbf{new}\, a)P) &= \{a\} \cup \mathsf{Act}(P) \\
\mathsf{Act}(P \mid Q) &= \mathsf{Act}(P) \cup \mathsf{Act}(Q) \\
\mathsf{Act}(P + Q) &= \mathsf{Act}(P) \cup \mathsf{Act}(Q)
\end{aligned}$$

Actions, free and bound actions, and action substitution

Free and bound actions

- $\mathsf{fn}(P) = \mathsf{Act}(P) \setminus \mathsf{bn}(P)$
- $\mathsf{bn}(P) \subseteq \mathsf{Act}$ is a set inductively defined by the rules

$$
\begin{aligned}
\mathsf{bn}(A\langle a_1, \ldots, a_n \rangle) &= \emptyset \\
\mathsf{bn}(\alpha.P) &= \mathsf{bn}(P) \\
\mathsf{bn}((\mathbf{new}\, a)P) &= \{a\} \cup \mathsf{bn}(P) \\
\mathsf{bn}(P \mid Q) &= \mathsf{bn}(P) \cup \mathsf{bn}(Q) \\
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\end{aligned}
$$

Actions, free and bound actions, and action substitution

Substitution

Let $P\{\vec{a} \leftarrow \vec{b}\}$ denote the simultaneous substitution of the free occurrences of the actions $\vec{a}$ in P for $\vec{b}$.

Example

$$
\begin{aligned}
\{water \leftarrow coffee\}\{cola \leftarrow tea\}(tea.\overline{pick}.VM + coffee.\overline{pick}.VM) \\
\{water \leftarrow coffee\}(cola.\overline{pick}.VM + coffee.\overline{pick}.VM) \\
(cola.\overline{pick}.VM + water.\overline{pick}.VM)
\end{aligned}
$$

Actions, free and bound actions, and action substitution

It is sometimes necessary to rename bound actions to avoid clashes.

$$((\textbf{new}\, a)a.b.0)\{a \leftarrow b\} = (\textbf{new}\, a)a.a.0$$

The free action b became a, which is bound...

Alpha-congruence

The binary relation $=_\alpha$ on processes is inductively defined by the rule $(\textbf{new}\, a)P =_\alpha (\textbf{new}\, b)P\{a \leftarrow b\}$ if $b \notin \text{bn}(P)$, and homomorphic rules on the remaining process constructs.

Structural Operational Semantics of CCS

- Syntax-driven proof rules to infer the behaviour of a system (Gordon Plotkin, 1981)
- Rules describe single computational steps, explaining the effect of executing a particular (syntactic) construct of the language

Transition Relation

Given a set of CCS defining equations $\mathcal{P}$ specifying a system, the transition relation of the system is defined by a set of triples

$$\{\xrightarrow{a} \in \mathcal{P} \times \mathcal{P} \mid a \in \mathsf{Act}\}$$

In the next slide we inductively define the Structural Operational Semantics of CCS

SOS proof rules of CCS

$$\frac{P_A\{\vec{a}\leftarrow\vec{b}\} \xrightarrow{\alpha} P'}{A\langle\vec{b}\rangle \xrightarrow{\alpha} P'}\ A(\vec{a}) \stackrel{\text{def}}{=} P_A\ \text{[Def]} \qquad \frac{}{\alpha.P \xrightarrow{\alpha} P}\ \text{[Pre]}$$

$$\frac{P \xrightarrow{\alpha} P'}{(\textbf{new}\,a)P \xrightarrow{\alpha} (\textbf{new}\,a)P'}\ \alpha \notin \{a, \overline{a}\}\ \text{[Res]} \qquad \frac{P \xrightarrow{\alpha} P'}{Q \xrightarrow{\alpha} P'}\ P =_\alpha Q\ \text{[Alpha]}$$

$$\frac{Q \xrightarrow{\alpha} Q'}{(Q \mid P) \xrightarrow{\alpha} (Q' \mid P)}\ \text{[L-Par]} \qquad \frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'}\ \text{[L-Sum]}$$

$$\frac{Q \xrightarrow{a} Q' \quad P \xrightarrow{\overline{a}} P'}{(Q \mid P) \xrightarrow{\tau} (Q' \mid P')}\ \text{[L-Sync]}$$

Par, Sum and Sync have right rules.

Running one example

Recall that

$$Serve = (tea.\overline{pick}.VM + coffee.\overline{pick}.VM)$$

and $VM \mid Client =$

$$coin.\overline{sugar}.(yes.fill.Serve + no.Serve)|\overline{coin}.sugar.\overline{no}.\overline{coffee}.pick.0$$

So,

$$\begin{aligned}
VM|Client &\xrightarrow{\tau} \overline{sugar}.(yes.fill.Serve + no.Serve)|sugar.\overline{no}.\overline{coffee}.pick.0\\
&\xrightarrow{\tau} (yes.fill.Serve + no.Serve) \mid \overline{no}.\overline{coffee}.pick.0\\
&\xrightarrow{\tau} Serve \mid \overline{coffee}.pick.0\\
&\xrightarrow{\tau} \overline{pick}.VM \mid pick.0\\
&\xrightarrow{\tau} VM \mid 0
\end{aligned}$$

How to justify each step?

Steps 4 and 5 are direct applications of the [Sync] rule.

Step 1

Let $Decide = \overline{sugar}.(yes.fill.Serve + no.Serve)$ and
$BCoffee = sugar.\overline{no}.\overline{coffee}.pick.0$

$$
\dfrac{
\dfrac{\dfrac{}{coin.Decide \xrightarrow{coin} Decide}\,[\text{Pre}]}{VM \xrightarrow{coin} Decide}\,[\text{Def}]
\qquad
\dfrac{\dfrac{}{coin.BCoffee \xrightarrow{\overline{coin}} BCoffee}\,[\text{Pre}]}{Client \xrightarrow{\overline{coin}} BCoffee}\,[\text{Def}]
}{VM \mid Client \xrightarrow{\tau} Decide \mid BCoffee}\,[\text{Sync}]
$$

Step 2 is similar to step 1

How to justify each step?

Step 3

Let $\mathit{Coffee} = \overline{\mathit{coffee}}.\mathit{pick}.0$

$$\dfrac{\dfrac{\dfrac{}{\mathit{no}.\mathit{Serve} \xrightarrow{\mathit{no}} \mathit{Serve}}\,[\mathrm{Pre}]}{\mathit{yes}.\mathit{fill}.\mathit{Serve} + \mathit{no}.\mathit{Serve} \xrightarrow{\mathit{no}} \mathit{Serve}}\,[\mathrm{Sum}] \qquad \dfrac{}{\overline{\mathit{no}}.\mathit{Coffee} \xrightarrow{\overline{\mathit{no}}} \mathit{Coffee}}\,[\mathrm{Pre}]}{\mathit{yes}.\mathit{fill}.\mathit{Serve} + \mathit{no}.\mathit{Serve} \mid \overline{\mathit{no}}.\mathit{Coffee} \xrightarrow{\tau} \mathit{Serve} \mid \mathit{Coffee}}\,[\mathrm{Sync}]$$