(1)

$\boxed{\text{Linear Temporal Logic}}$ (propositional)

Amir Pnueli (1970ies late)
$\Diamond$ "eventually"
$\Box$ "always"

relative order of eve[nts]
¬precise timi[ng]
time abstrac[tion]

Syntax

$$\varphi ::= \underbrace{\text{true}}_{\text{redundant if } AP \neq \emptyset} \mid \overset{AP}{\underset{}{a}} \mid \varphi \wedge \varphi \mid \neg \varphi$$ constants and logical connective

$\mathrm{For}_{LTL}(A)$ $$\mid \underset{\text{next}}{\bigcirc \varphi} \mid \underset{\text{until}}{\varphi \, U \, \varphi}$$ temporal modalities

right associative

definable: false, $\to$, $\leftrightarrow$, $\oplus$ - exclusive or / parity

false ::= ¬ true

$\varphi_1 \to \varphi_2$ ::= $\neg\varphi_1 \vee \varphi_2$

$\varphi_1 \leftrightarrow \varphi_2 ::= (\varphi_1 \to \varphi_2) \wedge (\varphi_2 \to \varphi_1)$
$= (\neg\varphi_1 \vee \varphi_2) \wedge (\neg\varphi_2 \vee \varphi_1)$

$\varphi_1 \oplus \varphi_2 ::= (\varphi_1 \wedge \neg\varphi_2) \vee (\neg\varphi_1 \wedge \varphi_2)$

$\Diamond$, $\Box$
eventually, always

precedence: unary $\neg, \bigcirc$; binary U; $\wedge, \vee, \to$

$$\{\wedge, \vee, \to\} < \{U\} < \{\neg, \bigcirc\}$$

$\neg\varphi_1 \, U \bigcirc \varphi_2$ $(\neg\varphi_1) U (\bigcirc\varphi_2)$

$\varphi_1 U \varphi_2 U \varphi_3$ $\varphi_1 U (\varphi_2 U \varphi_3)$

Intuitive semantics

An LTL formula expresses a property of an infinite path $\sigma \in 2^{AP}$
(i.e. models of LTL formulas are infinite sequences of sets $\subseteq 2^{AP}$)

"a is true": $\overset{a}{\bullet_0} \to \overset{\text{don't care}}{\bullet_1} \to \overset{\text{don't care}}{\bullet_2} \to \overset{..}{\bullet_3} \to \overset{..}{\bullet_4} \to \overset{...}{\bullet_5} \to \overset{...}{\bullet_6} \to \dots$

"$\bigcirc a$ is true" (next): $\overset{\text{don't care}}{\bullet_0} \to \overset{a}{\bullet_1} \to \overset{\text{don't care}}{\bullet_2} \to \bullet_3 \to \bullet_4 \to \bullet_5 \to \bullet_6 \to \dots$

"$a \, U \, b$ is true" (until): $\overset{a\wedge\neg b}{\bullet_0} \to \overset{a\wedge\neg b}{\bullet} \to \overset{a\wedge\neg b}{\bullet} \to \overset{a\wedge\neg b}{\bullet} \to \overset{a\wedge\neg b}{\bullet} \to \overset{b}{\bullet} \to \overset{\text{don't care}}{\bullet}$

"$\Diamond a$ is true" (eventually): $\overset{\neg a}{\bullet} \to \overset{\neg a}{\bullet} \to \overset{\neg a}{\bullet} \to \overset{\neg a}{\bullet} \to \overset{a}{\bullet} \to \overset{\text{don't care}}{\bullet} \to \overset{...}{\bullet} \to \dots$

"$\Box a$ is true" (always): $\overset{a}{\bullet} \to \overset{a}{\bullet} \to \overset{a}{\bullet} \to \overset{a}{\bullet} \to \overset{a}{\bullet} \to \overset{a}{\bullet} \to \overset{a}{\bullet} \to \dots$

Formal semantics

Let $\sigma \in (2^{AP})^\omega$ (= Words) and $\sigma = A_0 \dots A_i A_{i+1}$ then $\sigma_{\geq i} = A_i A_{i+1} \dots$, $\sigma[i] = A_i$

$\sigma \models_{LTL} \varphi$ ("σ models φ") if the statement "$\sigma \models \varphi$" follows from the following clauses:

$\sigma \models$ true
$\sigma \models a$ iff $a \in \sigma[0]$
$\sigma \models \varphi_1 \wedge \varphi_2$ iff $\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
$\sigma \models \neg\varphi$ iff $\sigma \not\models \varphi$ (that is, $\sigma \models \varphi$ does not hold)
$\sigma \models \bigcirc\varphi$ iff $\sigma_{\geq 1} \models \varphi$
$\sigma \models \varphi U \psi$ iff $\exists i \geq 0: \sigma_{\geq i} \models \psi$ and $\forall 0 \leq j < i: \sigma_{\geq j} \models \varphi$
$\sigma \models \Diamond\varphi$ iff $\exists i \geq 0: \sigma_{\geq i} \models \varphi$
$\sigma \models \Box\varphi$ iff $\forall i \geq 0: \sigma_{\geq i} \models \varphi$

$Words ::= (2^{AP})^\omega$

$Words(\varphi) := \{\sigma \in Words \mid \sigma \models \varphi\}$

Derived Modalities

"eventually" $\Diamond\varphi ::= true \; U \varphi$

$\sigma \models true \; U \varphi$ iff $\exists i \geq 0: \sigma_{\geq i} \models \varphi$ and $\forall 0 \leq j < i: \sigma_{\geq j} \models true$

iff $\exists i \geq 0: \sigma_{\geq i} \models \varphi$

iff $\sigma \models \Diamond\varphi$

"always" $\Box\varphi ::= \neg\Diamond\neg\varphi$

$= \neg(true \; U \neg\varphi)$

$\sigma \models \neg(true \; U \neg\varphi)$ iff $\sigma \not\models true \; U \neg\varphi$

iff $\neg\exists i \geq 0: \sigma_{\geq i} \models \neg\varphi$ and $\forall 0 \leq j < i: \sigma_{\geq j} \models true$

iff $\neg\exists i \geq 0: \sigma_{\geq i} \not\models \varphi$

iff $\forall i \geq 0: \sigma_{\geq i} \models \varphi$

iff $\sigma \models \Box\varphi$

$\sigma \models \neg\Diamond\neg\varphi$ iff $\sigma \not\models \Diamond\neg\varphi$

iff not $(\sigma \models \Diamond\neg\varphi)$

iff not $(\exists i \geq 0: \sigma_{\geq i} \models \neg\varphi)$

iff not $(\exists i \geq 0: \sigma_{\geq i} \not\models \varphi)$

iff not $(\exists i \geq 0:$ not $\sigma_{\geq i} \models \varphi)$

iff $\forall i \geq 0: \sigma_{\geq i} \models \varphi$

iff $\sigma \models \Box\varphi$

<u>Exercise</u>: define $\Box\varphi$ see above

<u>Exercise</u>: define "infinitely often φ" $\Box\Diamond\varphi$

"eventually forever φ" $\Diamond\Box\varphi$

$\sigma \models \Box\Diamond\varphi$ iff $\forall i \geq 0: \sigma_{\geq i} \models \Diamond\varphi$

iff $\forall i \geq 0: \exists j \geq i: \sigma_{\geq j} \models \varphi$ iff $\overset{\infty}{\exists} j: \sigma_{\geq j} \models$

$\sigma \models \Diamond\Box\varphi$ iff $\exists i \geq 0: \sigma_{\geq i} \models \Box\varphi$

iff $\exists i \geq 0: \forall j \geq i: \sigma_{\geq j} \models \varphi$ iff $\overset{\infty}{\forall} j: \sigma_{\geq j} \models$

Exercises: Which of the following equivalences are correct?

$\boxed{5}$ a) $\Box(\varphi \to \Diamond\psi) \equiv \varphi\, U\,(\psi \wedge \neg\varphi)$ $(\times)$

Countermodel: let $\psi := \varphi := p$ and $\sigma \not\models p$ for all $i \in \mathbb{N}$ (at $\sigma_{\geq i}$)

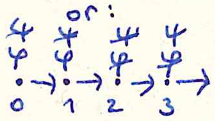
then $\sigma_{\geq i} \models \varphi \to \Diamond\psi$

and hence $\sigma \models \Box(\varphi \to \Diamond\psi)$

but $\sigma \not\models \varphi\, U\,(\psi \wedge \neg\varphi)$

because $\sigma_{\geq i} \not\models \psi \wedge \neg\varphi$ due to $\sigma_{\geq i} \not\models \psi$

or:



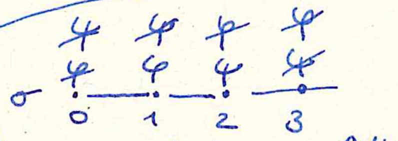
$\Box(\varphi \to \Diamond\psi)$

~~$\varphi\, U (\psi \wedge \neg\varphi)$~~

$\boxed{3}$ b) $\bigcirc\Diamond\varphi \equiv \Diamond\bigcirc\varphi$ $(\checkmark)$

$\boxed{4}$ c) $\Box(\varphi \wedge \bigcirc\Diamond\varphi) \equiv \Box\varphi$ $(\checkmark)$

$\boxed{2}$ d) $\Diamond(\varphi \wedge \psi) \equiv \Diamond\varphi \wedge \Diamond\psi$ $(\times)$



$\sigma \models \Diamond\varphi$, $\sigma \models \Diamond\psi$

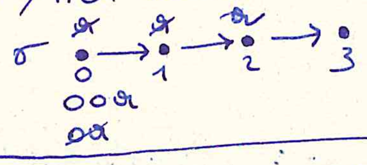
$\sigma \not\models \Diamond(\varphi \wedge \psi)$

$\boxed{7}$ e) $\Box(\varphi \wedge \psi) \equiv \Box\varphi \wedge \Box\psi$ $(\checkmark)$

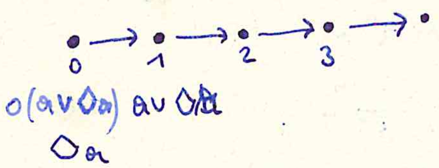
$\boxed{6}$ f) $\Box\Box(\varphi \to \psi) \equiv \neg\Diamond(\neg\psi \wedge \varphi)$ $(\checkmark)$

Ex. 5.24: Check the following LTL-formulas whether they are (i) satisfiable, and/or (ii) valid.

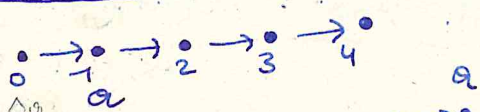
(a) $\bigcirc\bigcirc a \to \bigcirc a$ satisfiable / not valid



(b) $\bigcirc(a \vee \Diamond a) \to \Diamond a$ valid



Case 1:



Case 2:

(c) $\Box a \to \neg\bigcirc(\neg a \wedge \Box\neg a)$ (valid)

(d) $(\Box a)\, U\, (\Diamond b) \to \Box(a\, U\, \Diamond b)$ (valid)

(e) $\Diamond b \to a\, U\, b$ (satisfiable, but not valid)

$\varphi_1 \equiv \varphi_2$ equivalent $:\Leftrightarrow$ $Words(\varphi_1) = Words(\varphi_2)$

φ satisfiable $:\Leftrightarrow$ $Words(\varphi) \neq \emptyset$

φ valid $:\Leftrightarrow$ $Words(\varphi) = (2^{AP})^\omega$

Lemma: $\varphi_1 \equiv \varphi_2 \Leftrightarrow \varphi_1 \leftrightarrow \varphi_2$ valid

Recall $TS = \langle S, Act, \to, I, AP, L \rangle$ transition system.

states, actions, initial states, atomic propositions, $L: S \to 2^{AP}$ labeling function

$\to \subseteq S \times Act \times S$ transition relation

$\boxed{\text{we may assume all paths and traces are infinite}}$

$\pi \in Paths(TS)$: (infinite path segments) $\quad \pi \models \varphi :\Longleftrightarrow trace(\pi) \models \varphi$

$s \in S$: $\quad s \models \varphi :\Longleftrightarrow \forall \pi \in Paths(s) : \pi \models \varphi$

$TS \models \varphi :\Longleftrightarrow \forall s \in I : s \models \varphi$

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examples

A note on negation:

$Words(\varphi) = \{ \sigma \in (2^{AP})^\omega \mid \sigma \models \varphi \}$

$Words(\neg\varphi) = (2^{AP})^\omega \setminus Words(\varphi)$ due to $\sigma \models \varphi \Longleftrightarrow \sigma \not\models \neg\varphi$

$\sigma \not\models \varphi \Longleftrightarrow \sigma \models \neg\varphi$

$\boxed{\text{traces decide formulas}}$

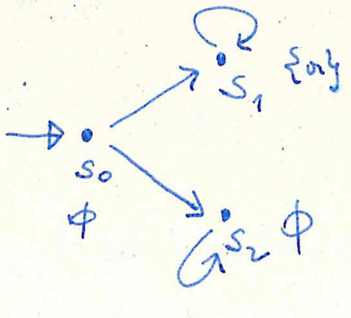
(an LTL formula is either true or false for a trace)

However: transition systems do not decide formulas

$TS \models \varphi \quad \overset{\Longrightarrow}{\not\Longleftarrow} \quad TS \not\models \neg\varphi$

$TS \not\models \varphi \quad \underset{\Longleftarrow}{\not\Longrightarrow} \quad TS \models \neg\varphi$

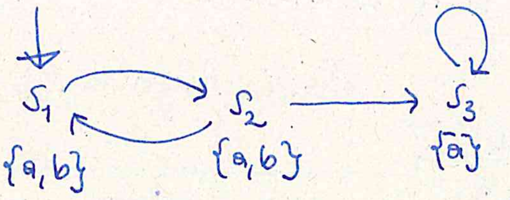
Take TS



$TS \not\models \Diamond a$ because $s_0 s_2 s_2 \ldots \not\models \Diamond a$

$TS \not\models \neg\Diamond a$ because $s_0 s_1 s_1 \ldots \not\models \neg\Diamond a$
due to $s_0 s_1 s_1 \ldots \models \Diamond a$

Exercise. Let $AP=\{a,b\}$ and TS =



a) Is TS deterministic? ~~no~~, neither action-deterministic nor AP-determi

b) $s_1 \models \bigcirc(a \wedge b)$? (✓)

c) $s_2 \models \bigcirc(a \wedge b)$? (×) because $s_2 s_3^\omega \not\models \bigcirc b$ and hence $s_2 s_3^\omega \not\models \bigcirc(a$

d) $TS \models \Box a$? (✓)

e) $TS \models \Box(\neg b \to a)$? (✓)

f) $TS \models \Diamond \neg b$? (×) since $s_1 s_2 s_1 s_2 \ldots \not\models$

$TS \models \Box b$ (×) since $s_1 s_2 s_3^\omega \not\models \Box b$

g) $TS \models \Diamond \Box \neg b$? (×)

$\text{Words}(\varphi) = \{\sigma \in (2^{AP})^\omega \mid \sigma \models \varphi\}$

π infinite path fragment of TS

$\pi \models \varphi :\Longleftrightarrow \text{trace}(\pi) \models \varphi$

$\Longleftrightarrow \text{trace}(\pi) \in \text{Words}(\varphi)$

For all states we can define

$s \models \varphi :\Longleftrightarrow \forall \pi \in \text{Paths}(s).\ \pi \models \varphi$ where $s \in S$

And finally

$TS \models \varphi :\Longleftrightarrow \forall s \in I: s \models \varphi$

Input: φ LTL formula & TS finite without terminal states.

Output: "yes" if $TS \models \varphi$, otherwise counterexample

$\mathcal{A}_{\neg\varphi} ::$ NBA s.th. $\mathcal{L}(\mathcal{A}_{\neg\varphi}) = \text{Words}(\neg\varphi)$ Thm. Büchi automata accept ω-reg. languages

state explosion $\mathcal{A} ::$ $TS \otimes \mathcal{A}_{\neg\varphi}$ emptiness-checking

if $\exists \pi \in \text{paths}(\mathcal{A}) : \pi$ satisfies the acceptance condition of $\mathcal{A}$

then return (expressive) bad prefix of π

else return "yes"

fi

NBA $\mathcal{A}$ has empty language

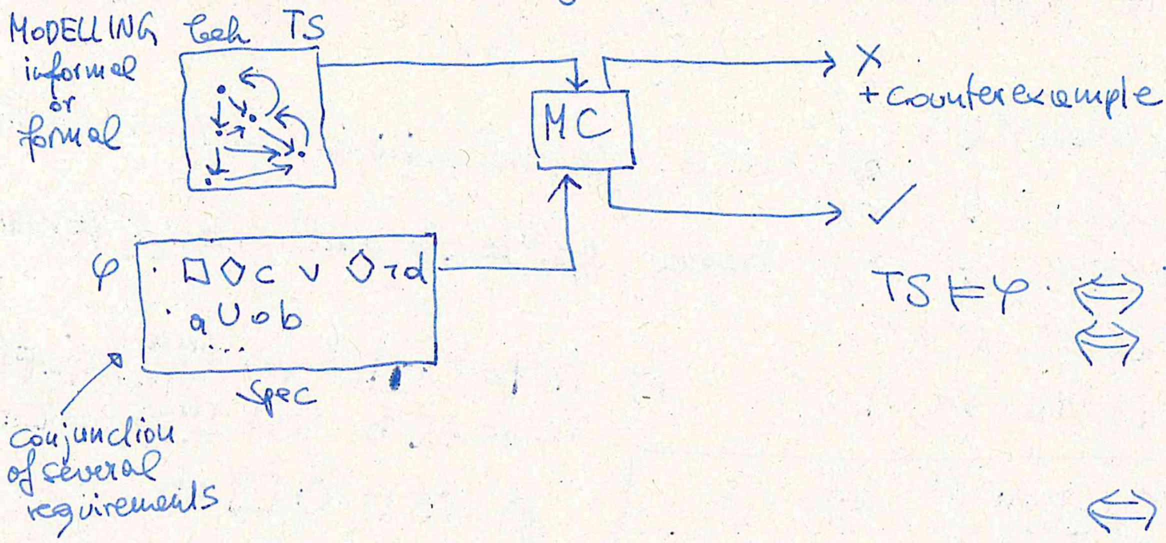
$\Updownarrow$

$\neg\exists$ reachable accepting state on a cycle

... is recognisable in linear time

Model checking in LTL

Basic Algorithm (Vardi, Wolper 1986)



$TS \not\models \varphi$

$\pi \not\models \varphi$

$TS \models \varphi$

$TS \models \varphi \iff Traces(TS) \subseteq Words(\varphi)$

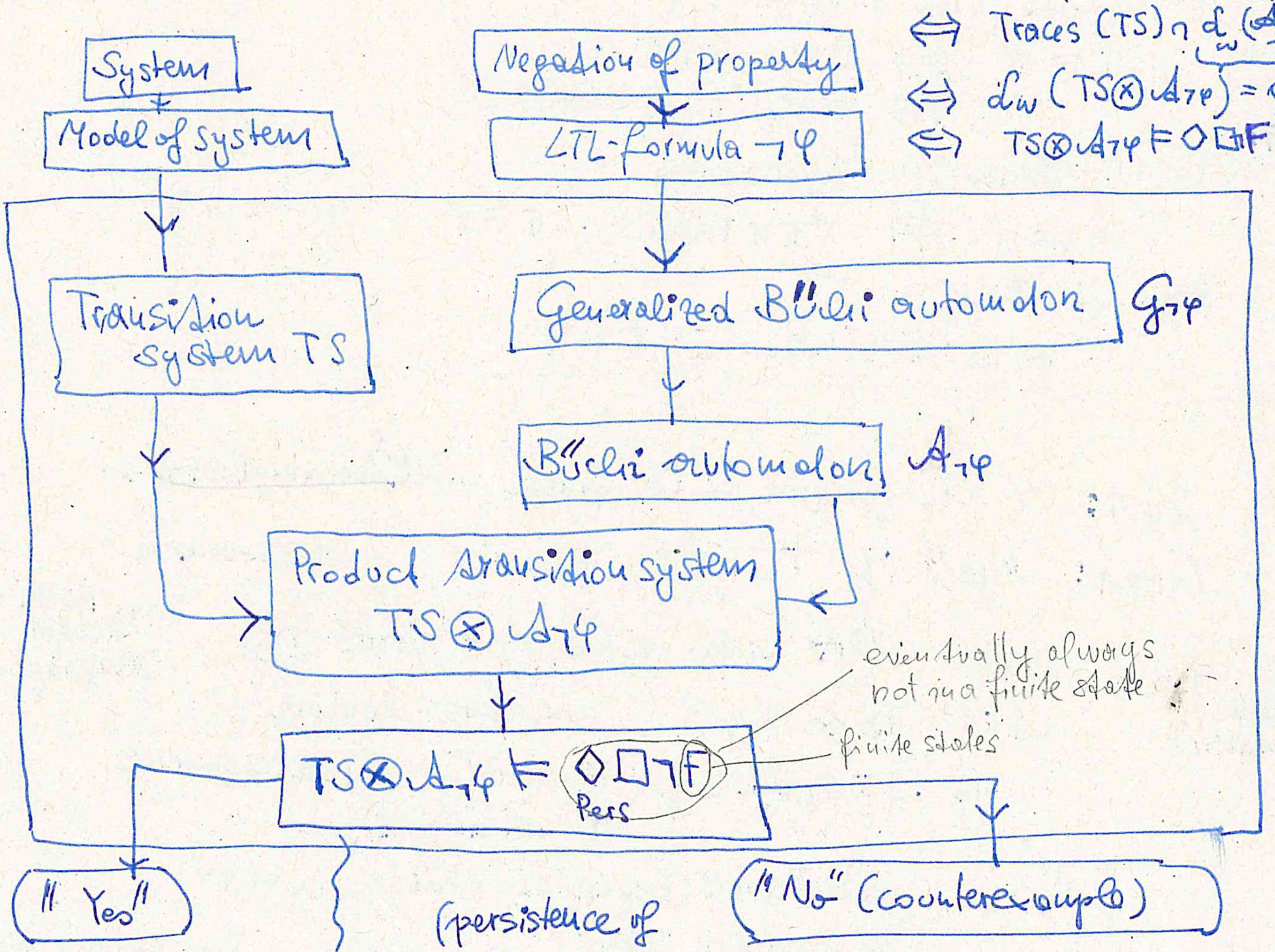
$\iff Traces(TS) \cap \left((2^{AP})^\omega \setminus Words(\varphi) \right) = \emptyset$

$\iff Traces(TS) \cap Words(\neg\varphi) = \emptyset$

$\iff Traces(TS) \cap \mathcal{L}_\omega(\mathcal{A}_{\neg\varphi}) = \emptyset$

$\iff \mathcal{L}_\omega(TS \otimes \mathcal{A}_{\neg\varphi}) = \emptyset$

$\iff TS \otimes \mathcal{A}_{\neg\varphi} \models \Diamond\Box\neg F$



(persistence of forever no accept state)

Check whether there is a reachable final state on a cycle of $TS \otimes \mathcal{A}_{\neg\varphi}$

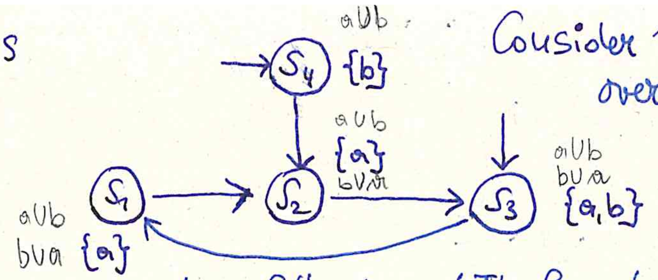
(can be checked in linear time)

Complexity $O(|TS| \cdot 2^{|\varphi|})$

PSPACE-Complete

5.5 Exercises

Exercise 5.1.



Consider the following transition system over set $\{a,b\}$ of atomic propositions:

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$J = \langle \{s_1, s_2, s_3, s_4\}, 2^{\{a,b\}}, \rightarrow, \{s_3, s_4\}, \{a,b\}, L \rangle$

Indicate for each of the following LTL-formulae the set of states for which these formulae are fulfilled:

(a) $\bigcirc a$ $\{s_1, s_2, s_3, s_4\} = \{s \in S / s \models_J \bigcirc a\}$

path

$\pi \models \varphi :\Leftrightarrow \text{trace}(\pi) \models \varphi$

$s \models \varphi :\Leftrightarrow \forall \pi \in \text{Paths}(s).\ \pi \models \varphi$

$TS \models \varphi :\Leftrightarrow (\forall \pi \in \text{Paths}(s_i), s_i$ initial state of $TS)\ \pi \models \varphi$

$\forall s_{i_{init}} \in TS$, s_i initial state: $s_{init} \models \varphi$

$\Leftrightarrow \text{Traces}(TS) \subseteq \text{Words}(\varphi)$

$\text{Words}(\varphi) := \{\sigma \in (2^{AP})^\omega / \sigma \models \varphi\}$

$\text{Traces}(TS) := \{\text{trace}(\pi) / \pi \in \text{Paths}_{TS}(s_{init}), s_{init} \in I\}$

$\pi = s_1 s_2 s_3 \cdots$

$\text{trace}(\pi) = L(s_1) L(s_2) L(s_3) \cdots$

$TS = \langle S, Act, \rightarrow, I, AP, L \rangle$

$\rightarrow \subseteq S \times Act \times S$ transition relation

$I \subseteq S$ initial states

AP atomic propositions

$L : S \rightarrow 2^{AP}$

(b) $\bigcirc\bigcirc\bigcirc a$ $\{s \in S \mid s \models \bigcirc\bigcirc\bigcirc a\} = \{s_1, s_2, s_3, s_4\}$

(c) $\square b$ $\{s \in S \mid s \models \square b\} = \{\}$

(d) $\square \lozenge a$ $\{s \in S \mid s \models \square \lozenge a\} = S$

(e) $\square (b \cup a)$ $\{s \in S / s \models \square (b \cup a)\} = S$

(f) $\lozenge (a \cup b)$ $\{s \in S / s \models \lozenge (a \cup b)\} = S$

(g) $\square a$ $\{s \in S / s \models \square a\} = \{s_1, s_2, s_3\}$

Exercise 5.2 Consider the transition system over the set of atomic propositions $AP = \{a, b, c\}$:

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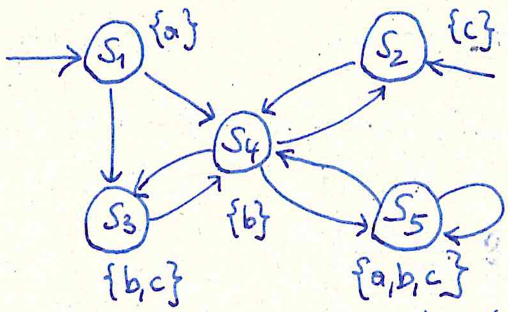


Decide for each LTL formulae φ_i below, whether $TS \models \varphi_i$ holds. Justify your answers. If $TS \not\models \varphi_i$ provide a path $\pi \in Paths(TS)$ such that $\pi \not\models \varphi_i$

$\varphi_1 = \Diamond \Box c$ $\quad TS \not\models \varphi_1 \quad s_1 s_3 s_4 s_3 s_4 \ldots \not\models \Diamond \Box c$

$\varphi_2 = \Box \Diamond c$ $\quad TS \models \varphi_2$

$\varphi_3 = \bigcirc \neg c \rightarrow \bigcirc \bigcirc c$ $\quad TS \models \varphi_3$

$\varphi_4 = \Box a$ $\quad TS \not\models \varphi_4 \quad s_2 s_4 \ldots \not\models a$

hence $s_2 s_4 \ldots \not\models \Box a$

$\varphi_5 = a \, U \, \Box (b \vee c)$ $\quad TS \models \varphi_5$

$$\left.\begin{array}{l} s_2, \ldots, s_5 \models \Box b \vee c \\ s_1 \models a \end{array}\right\} \Rightarrow \begin{array}{l} s_2 \models a \, U \, (b \vee c) \\ s_1 \models a \, U \, (b \vee c) \\ \Rightarrow TS \models \varphi_5 \end{array}$$

$\varphi_6 = (\bigcirc \bigcirc b) \, U \, (b \vee c)$ $\quad TS \not\models \varphi_6$ because $s_1 s_4 s_2 \ldots \not\models \bigcirc \bigcirc b$

$\not\models b \vee c$

$\varphi_1' = \Diamond \Box b$ $\quad TS \not\models \varphi_1'$

$\varphi_2' = \Box \Diamond b$ $\quad TS \models \varphi_2'$

$TS = \langle S, Act, \rightarrow, I, AP, L \rangle$

$\pi \in Path(TS)$: $\quad \pi \models \varphi :\Leftrightarrow trace(\pi) \models \varphi$

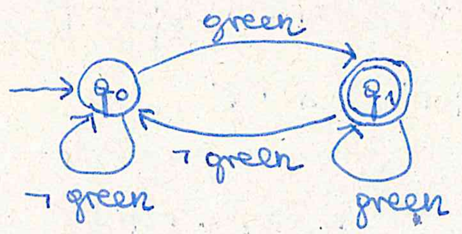
$s \in S$: $\quad s \models \varphi :\Leftrightarrow \forall \pi \in Path(s) : \pi \models \varphi$

$TS \models \varphi :\Leftrightarrow \forall s \in I . \; s \models \varphi$

$\sigma \models \varphi \, U \, \psi \Leftrightarrow \exists j \geq 0 : \sigma_j \models \psi$ and $\forall 0 \leq i < j : \sigma \models \varphi$

non-deterministic Büchi automaton

NBA for $\Box\Diamond$ green (infinitely often green)

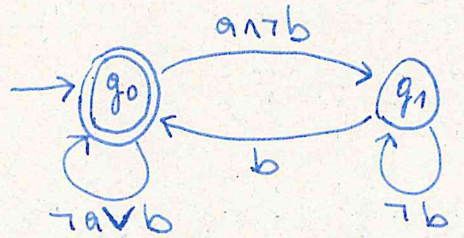


deterministic

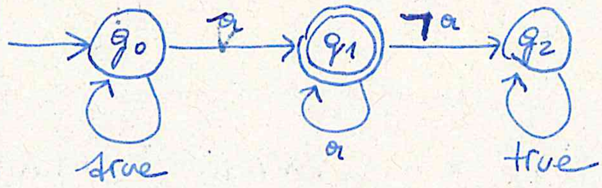
{green} $\emptyset$ {green} $\emptyset$ ...

q_0 q_1 q_0 q_1 infinitely often visits end state q_1 $\Downarrow$ is accepting computation.

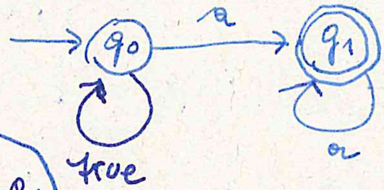
NBA for $\Box(a \to \Diamond b)$



NBA's for $\Diamond\Box a$ (almost always a)



Complete NBA



incomplete NBA $Q \times A \to \mathcal{P}(Q)$

(Q, A, δ, I, F)

$q_0 q_1 q_2 \ldots$

$\sigma \in (2^{AP})^\omega$ an accepting run of Büchi automaton $\mathcal{A}$ if $\overset{\infty}{\exists} j \geq 0:$ $q_j \in F$

$\sigma = A_0 A_1 A_2 A_3 \ldots$

Büchi-automata accept ω-regular languages.

Julius Richard Büchi (1924–1984)

$G = E_1 . F_1^\omega + \ldots + E_n . F_n^\omega$

where $E_1, \ldots, E_n, F_1, \ldots, F_n$ are reg. expressi

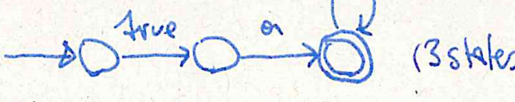
$E, E_1, E_2 ::= \emptyset \mid \hat{a} \mid E_1 + E_2 \mid E_1 . E_2 \mid E_1^*$

NBA for $\bigcirc a$

possible with 2 states?

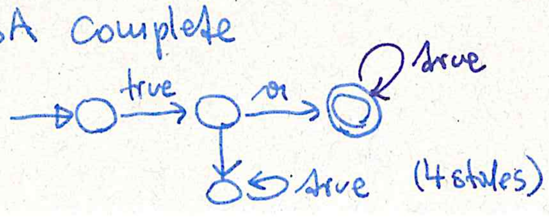
(probably still yes with 2 initial states)

DBA incomplete



(3 states)

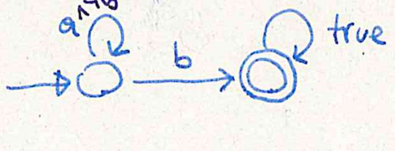
DBA complete



(4 states)

DBA complete

NBA for $a \mathsf{U} b$



(2 states)

Baier/Katoen, p 285:

"For example, for the LTL formulae $\bigcirc a$ and $a \mathsf{U} b$ an NBA with 2 states suffices. (It is left to the reader to provide these NBAs.)"

Weak Until

$$\sigma \models \varphi W \psi \;:\Leftrightarrow\; \left(\exists i \geq 0 : \sigma_{\geq i} \models \psi \wedge \forall 0 \leq j < i .\ \sigma_{\geq j} \models \varphi\right)$$
$$\vee\ \forall i \geq 0 : \sigma_{\geq i} \models \varphi \wedge \neg\psi$$

$$\neg(\varphi U \psi) \equiv \underbrace{(\varphi \wedge \neg\psi) U (\neg\varphi \wedge \neg\psi) \ \vee\ \Box(\varphi \wedge \neg\psi)}_{(\varphi \wedge \neg\psi) W (\neg\varphi \wedge \neg\psi)}$$

motivation.

Def. $\varphi W \psi := (\varphi U \psi) \vee \Box\varphi$

Then $\neg(\varphi U \psi) \equiv (\varphi \wedge \neg\psi) W (\neg\varphi \wedge \neg\psi)$

$\neg(\varphi W \psi) \equiv (\varphi \wedge \neg\psi) U (\neg\varphi \wedge \neg\psi)$

Positive Normal Form for LTL (Weak-Until PNF)

$$\varphi ::= true / false \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 U \varphi_2 \mid \varphi_1 W \varphi_2$$

Fairness in LTL

Definition. Let Φ and Ψ be propositional logic formulae over AP.

1. An unconditional LTL fairness constraint is an LTL-formula of the fo[rm]

$$ufair = \square\lozenge\Psi$$

2. A strong LTL fairness condition:

$$sfair = \square\lozenge\Phi \rightarrow \square\lozenge\Psi.$$

3. A weak LTL-fairness condition:

$$wfair = \lozenge\square\Phi \rightarrow \square\lozenge\Psi.$$

"$\rho = s_0 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_3} \ldots$"
ρ is unconditionally A-fair
if $\overset{\infty}{\exists} j \geq 0 : \alpha_j \in A$

ρ is strongly A-fair
if $\overset{\infty}{\exists} j \geq 0.\ A \cap Act(s_j) \neq \emptyset$
$\Rightarrow \overset{\infty}{\exists} j \geq 0 : \alpha_j \in A$

weakly A-fair
if $\overset{\infty}{\forall} j \geq 0 : A \cap Act(s_j) \neq \emptyset$
$\Rightarrow \overset{\infty}{\exists} j \geq 0 : \alpha_j \in A$

An LTL fairness assumption is a conjunction of LTL fairness constraints of arbitrary type

$$sfair = \bigwedge_{0 < i \leq k} (\square\lozenge\Phi_i \rightarrow \square\lozenge\Psi_i)$$

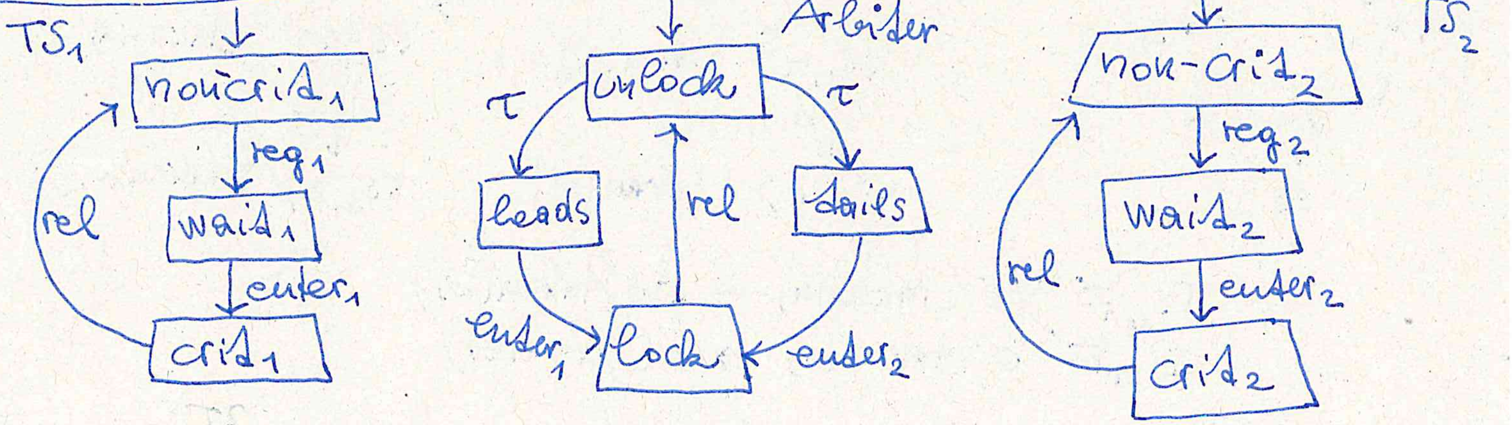
$$fair = ufair \wedge sfair \wedge wfair.$$

$$FairPaths(s) = \{\pi \in Paths(s) \mid \pi \models fair\}$$

$$FairTraces(s) = \{trace(\pi) \mid \pi \in FairPaths(s)\}$$

$$s \models_{fair} \varphi \quad :\Longleftrightarrow \quad \forall \pi \in FairPaths(s) : \pi \models \varphi$$

$$TS \models_{fair} \varphi \quad :\Longleftrightarrow \quad \forall s_0 \in I : s_0 \models_{fair} \varphi$$

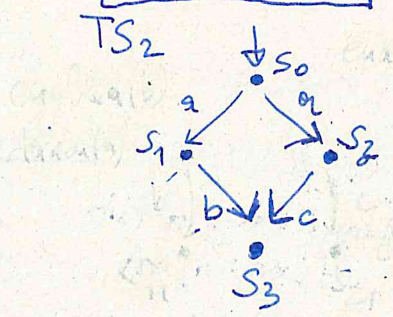


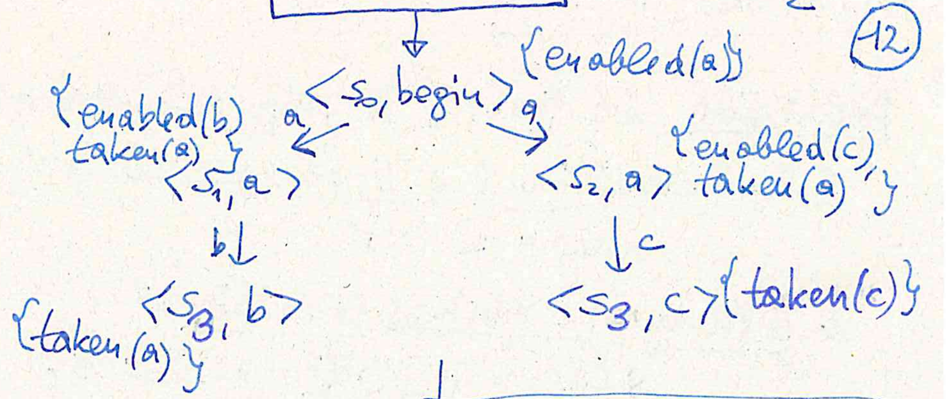
$$TS_1 \parallel Arbiter \parallel TS_2 \not\models \square\lozenge crit_1$$

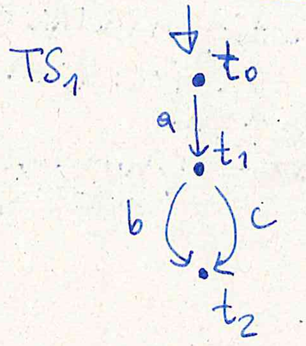
$$fair = \square\lozenge heads \wedge \square\lozenge tails$$

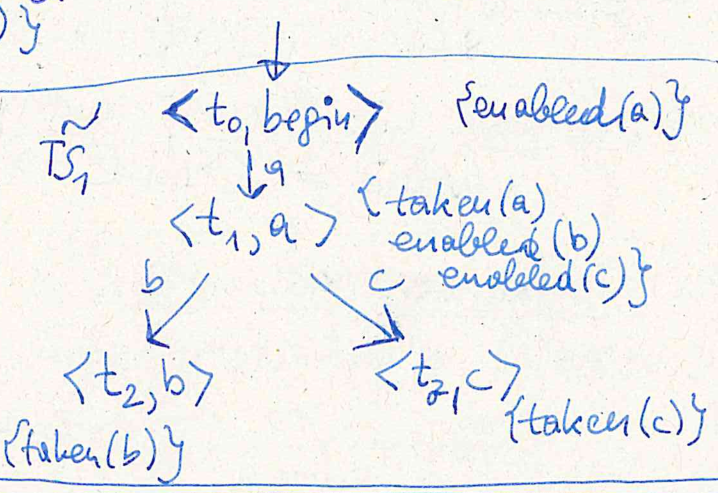
$$TS_1 \parallel Arbiter \parallel TS_2 \models_{fair} \square\lozenge crit_1 \wedge \square\lozenge crit_2$$

action-based versus **state-based**





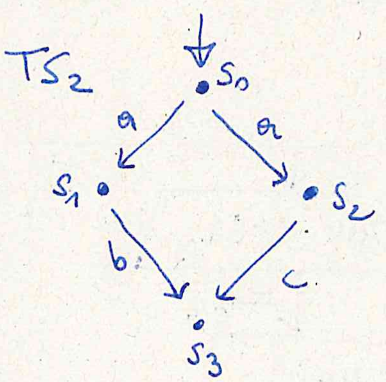


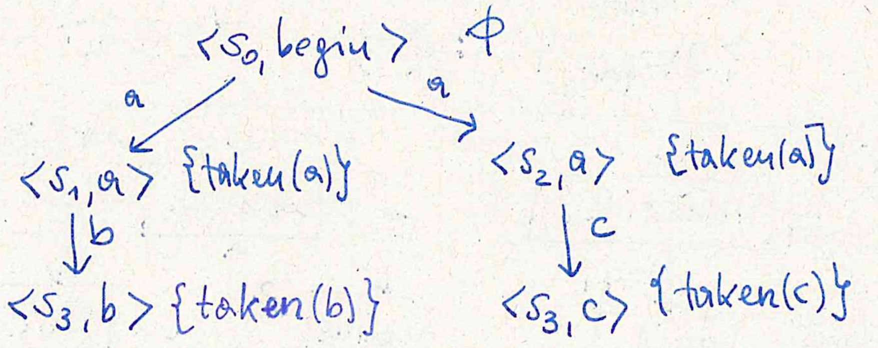


teaser for Lecture on CTL

$\varphi \equiv \forall \circ\, enabled(b)$ $\qquad \varphi \equiv \exists \circ\, enabled$

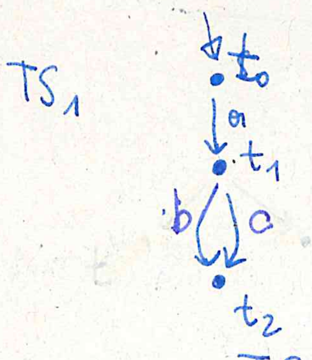
$\widetilde{TS}_1 \models \varphi \qquad \widetilde{TS}_2 \not\models \varphi$

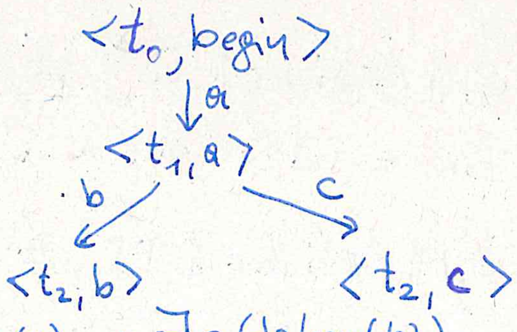




$TS_2 \not\models \forall \circ (taken(a) \rightarrow \exists \circ (taken(b))$

$TS_2 \not\models \circ (taken(a) \rightarrow \circ (taken(b))$





$TS_1 \models \forall \circ (taken(a) \longrightarrow \exists \circ (taken(b))$

$TS_1 \not\models \circ (taken(a) \longrightarrow \circ (taken(b))$