Another (important) variant of TS

A **labelled transition system** is a triple $(S, A, \rightarrow)$ where

- S is a set of states
- A is a set of labels (or actions, or operations, or events, ...)
- $\rightarrow \subseteq S \times A \times S$ $\quad (\rightarrow : S \rightarrow 2^{A \times S})$ transition relation

$$s \xrightarrow{a} s'$$

observable information about what happens during the transition

particularly handy to model communication / concurrency / distribution

Example An FSA $M = (Q, \Sigma, q_0, \delta, F)$ is an LTS:

$LTS_M = (S \cup \{\bullet\}, \Sigma \cup \{\checkmark\}, \rightarrow)$ where $s \xrightarrow{a} s' \iff a \in \Sigma \;\&\; s' \in \delta(s,a)$ or $a = \checkmark \;\&\; s' = \bullet \;\&\; s \in F$

$$\mathcal{L}_M = \{a_1 .. a_n \in \Sigma^* \mid \exists q_1, .., q_n \mid q_0 \xrightarrow{a_1} \ldots q_{n-1} \xrightarrow{a_n} q_n \xrightarrow{\checkmark} \bullet\}$$

Communication-based concurrency

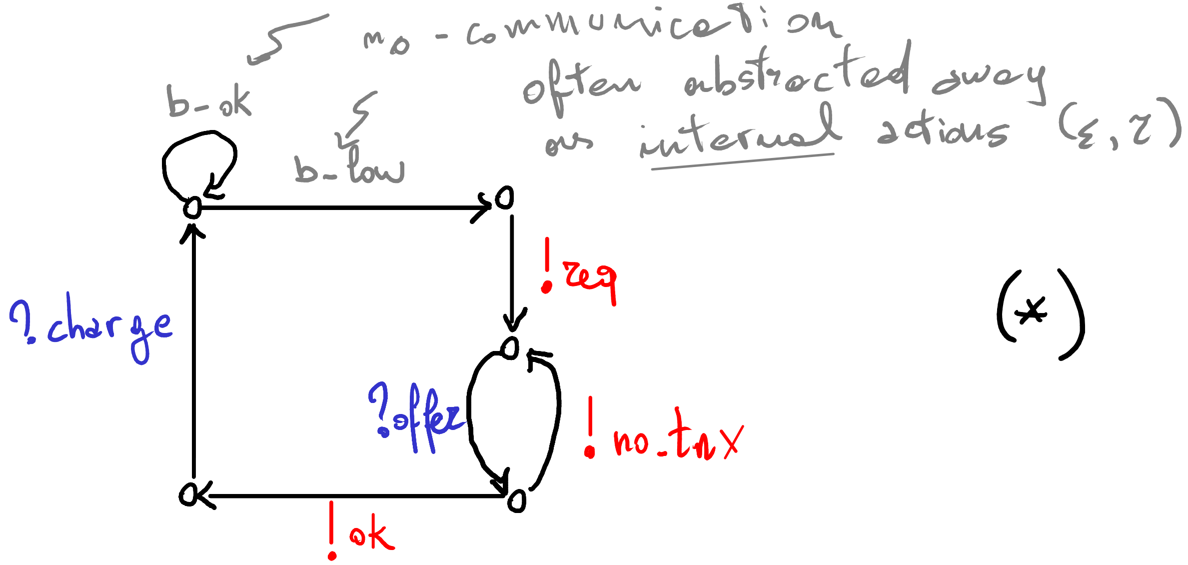
A robotic scenario:
Some mobile robots need to manage their energy in order to accoplish their task (e.g., patrolling some premises).

- When their batteries deplete, robots look for a recharge.
- Recharges are offered by recharge stations or other robots.

We can model this behaviour using an LTS capturing the observable features we are interested in: in this case communication
For instance, the behaviour of a robot seeking for a recharge is

The set of labels is the union of

- {b_low, b_ok} internal actions
- {?charge, ?offer} input actions
- {!req, !no_tnx} output actions



Exercise 4
Give an LTS modelling the behaviour of a robot offering a recharge.
Reflect about the "compatibility" between your solution and the LTS (*) above

Regular expressions

BNF-like syntax $\quad$ A, finite alphabet

$$E ::= 0 \mid 1 \mid a \mid E+E \mid E \cdot E \mid E^*$$

end $\quad$ skip $\quad$ atomic instruct. $\quad$ if-then-else $\quad$ -;- $\quad$ iteration

Denotational semantics : $\mathcal{L} : E \longrightarrow 2^{A^*}$ $\quad$ Term-Algebra homomorphism

$$\mathcal{L}(0) = \emptyset \qquad \mathcal{L}(1) = \{\varepsilon\} \qquad \mathcal{L}(a) = \{a\}$$

$$\mathcal{L}(E_1 + E_2) = \mathcal{L}(E_1) \cup \mathcal{L}(E_2)$$

$$\mathcal{L}(E_1 \cdot E_2) = \mathcal{L}(E_1) \cdot \mathcal{L}(E_2) \triangleq \{ v w \in A^* \mid v \in \mathcal{L}(E_1),\ w \in \mathcal{L}(E_2) \}$$

$$\mathcal{L}(E^*) = \mathcal{L}(E)^* = \bigcup_{n \geq 0} \mathcal{L}(E)^n .$$

Exercise 6
Prove or disprove that (a + b)* = (a* + b*)*