# Formal Methods for Communication Protocols

## Harnessing distributed software design with behavioural contracts

Emilio Tuosto @ GSSI

– Lecture 2 –

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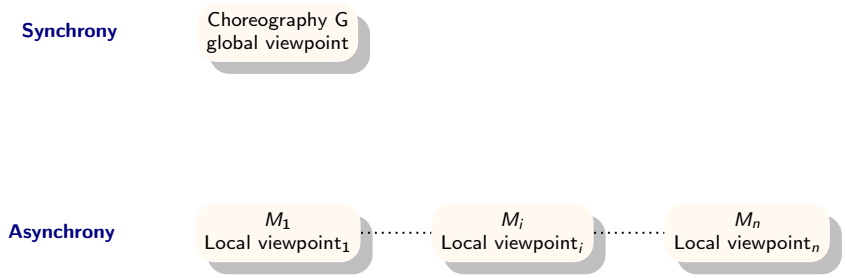
Finanziato dall'Unione europea NextGenerationEU | Ministero dell'Università e della Ricerca | Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA | DESK Developing Shared Knowledge

# – Choreographic Models –

# "Top-down"

## Quoting W3C [7]

"Using the Web Services Choreography specification, a contract containing a global definition of the common ordering conditions and constraints under which messages are exchanged, is produced that describes, from a global viewpoint [...] observable behaviour of all the parties involved. Each party can then use the global definition to build and test solutions that conform to it. The global specification is in turn realised by combination of the resulting local systems [...]"



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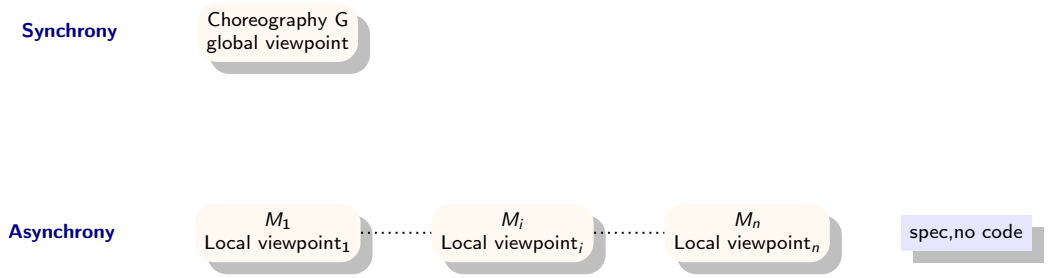

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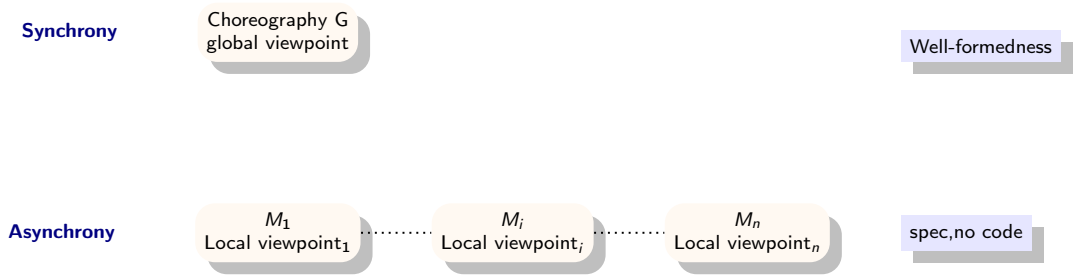

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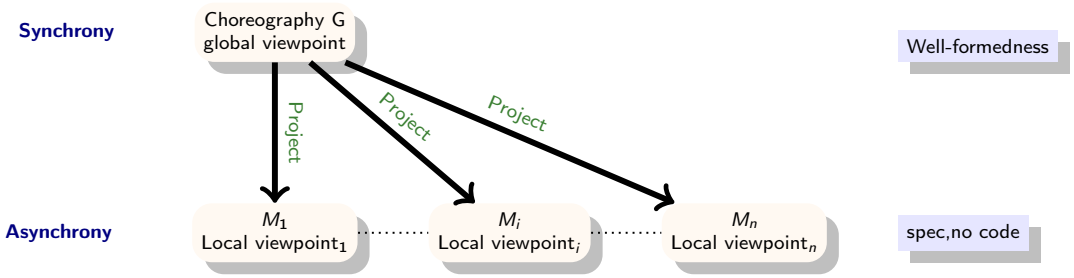

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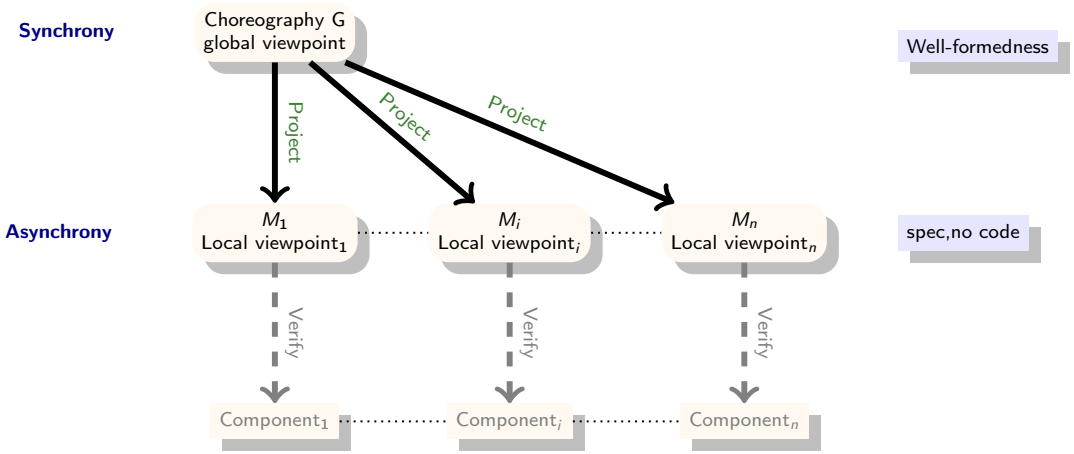
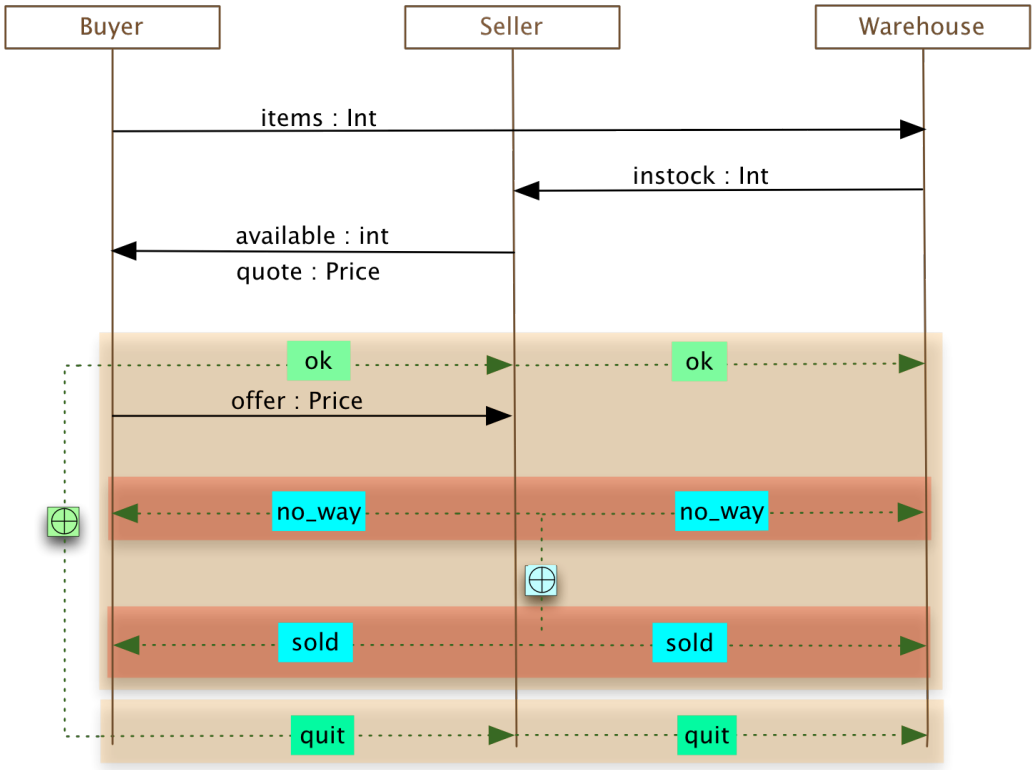


# An intuitive account...




Global viewpoint

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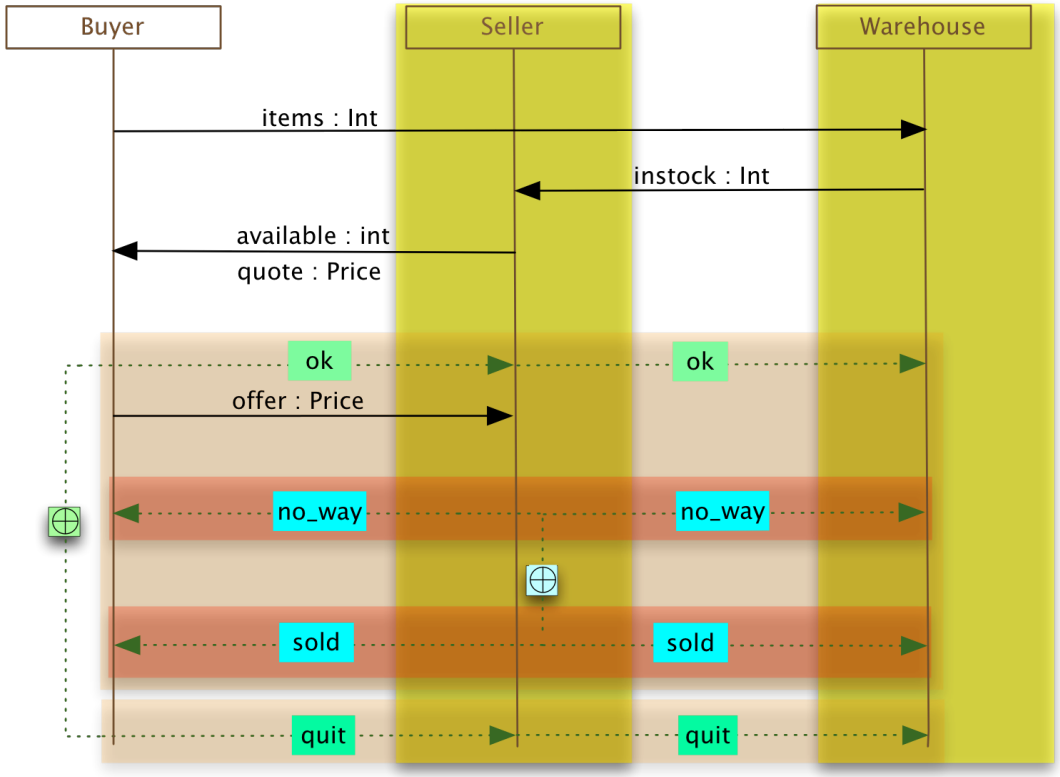


Projecting on buyer

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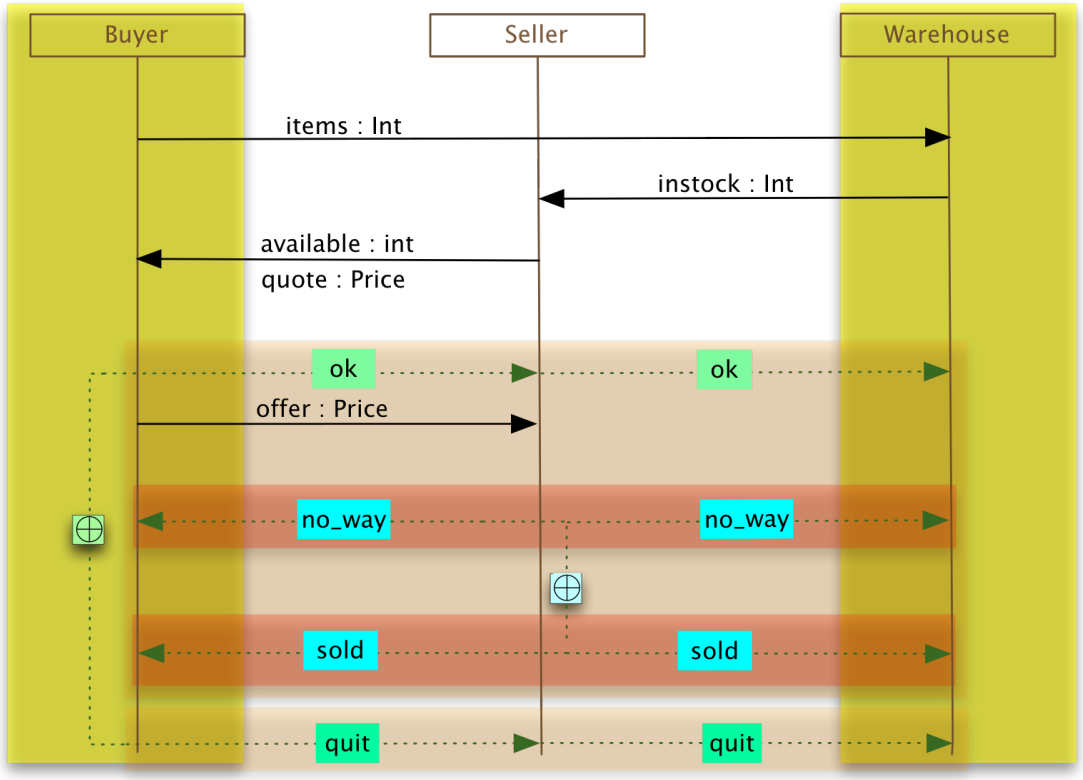


Projecting on seller

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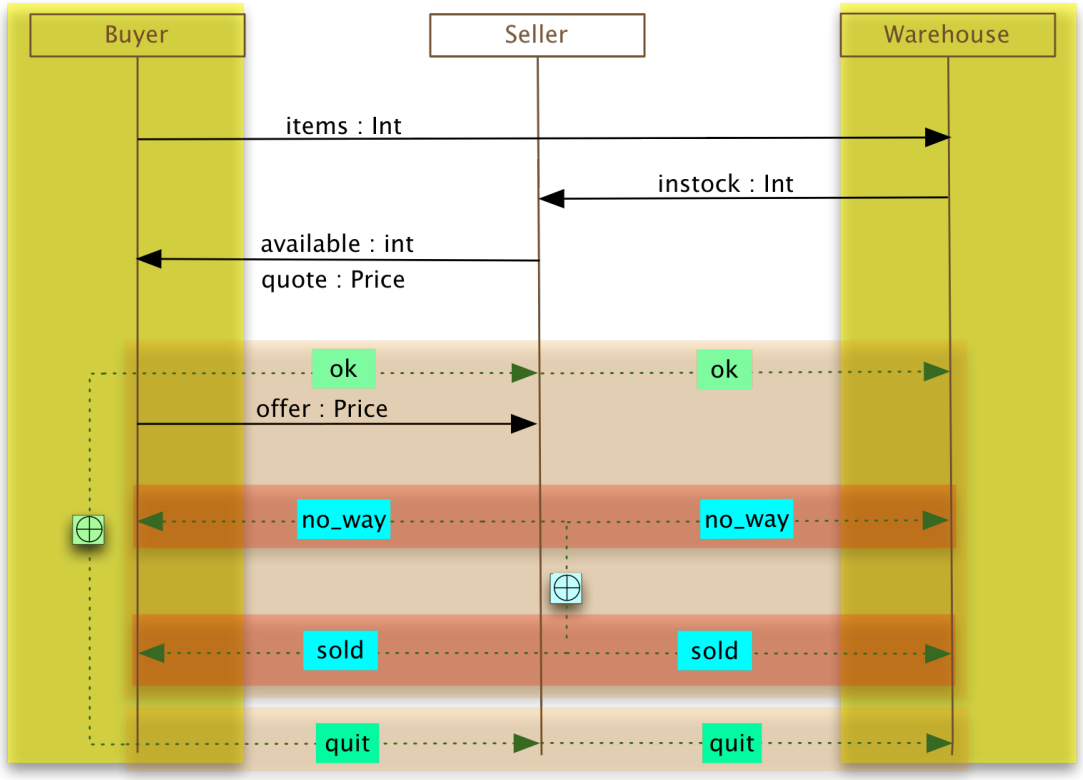


Projecting on seller

Life is harder...recall the bugs of pingpong

# Global views...a bit more formally

G-choreographies [9, 5]

$$\mathsf{G}, \mathsf{G}' \ ::= \ \circledcirc \ \mid \ \mathsf{A} \to \mathsf{B} : \mathsf{m} \ \mid \ \mathsf{G} \mid \mathsf{G}' \ \mid \ \mathsf{G}; \mathsf{G}' \ \mid \ \mathsf{G} + \mathsf{G}' \ \mid \ *\mathsf{G}$$

i.e., regular expressions (on an alphabet of interactions) with parallel composition

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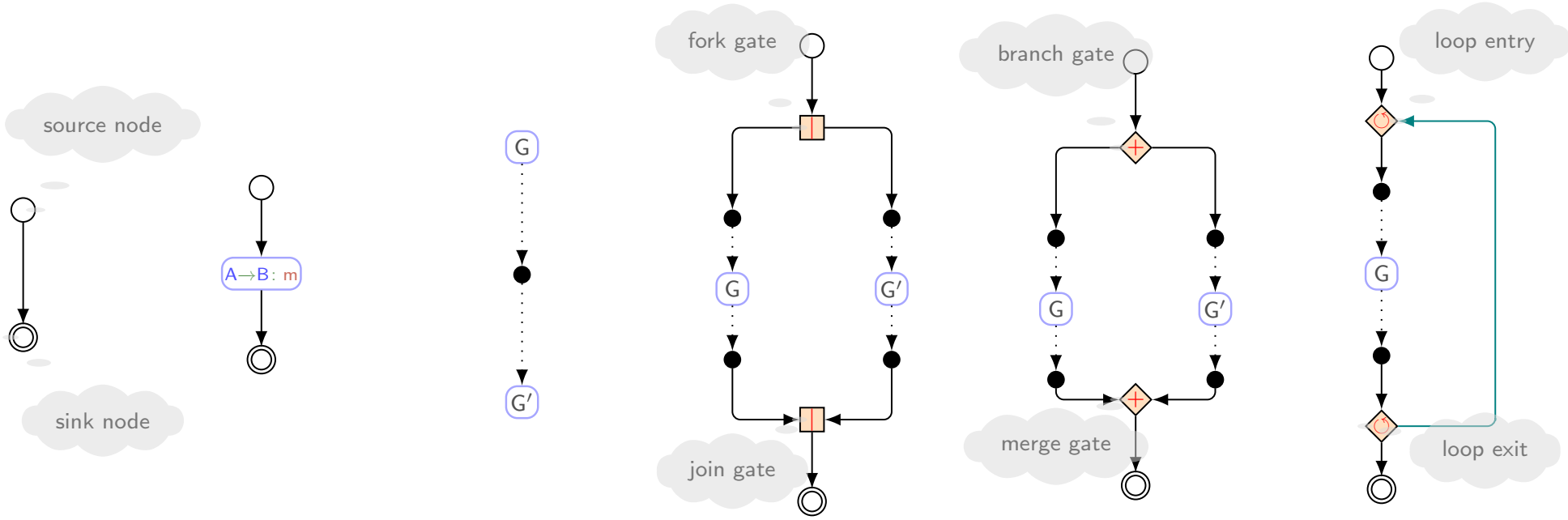


empty | interaction | sequential | parallel | branch | iteration

# Global views' semantics as pomsets

A **pomset** on a set $\mathcal{E}$ of **events** is (an isomorphism class of) a **labelled partially-order** $(\mathcal{E}, \lambda, \leq)$, with

- $\lambda : \mathcal{E} \to \mathcal{L}$ a labelling function
- $\leq \quad \subseteq \quad \mathcal{E} \times \mathcal{E}$ reflexive, anti-symmetric, transitive

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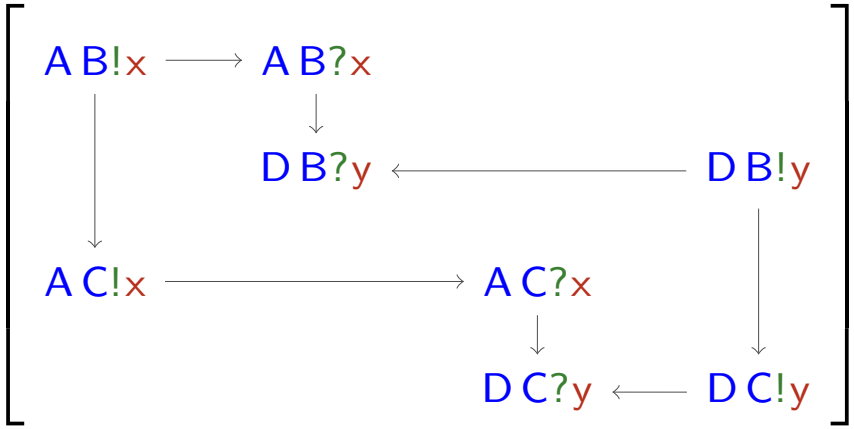
**Communication events**: $\mathcal{E} = \mathcal{E}^! \cup \mathcal{E}^?$ where

$$\begin{aligned} \mathcal{E}^! &= \mathcal{C} \times \{!\} \times \mathcal{M} \\ \mathcal{E}^? &= \mathcal{C} \times \{?\} \times \mathcal{M} \end{aligned}$$

# A simple pomset of communication events



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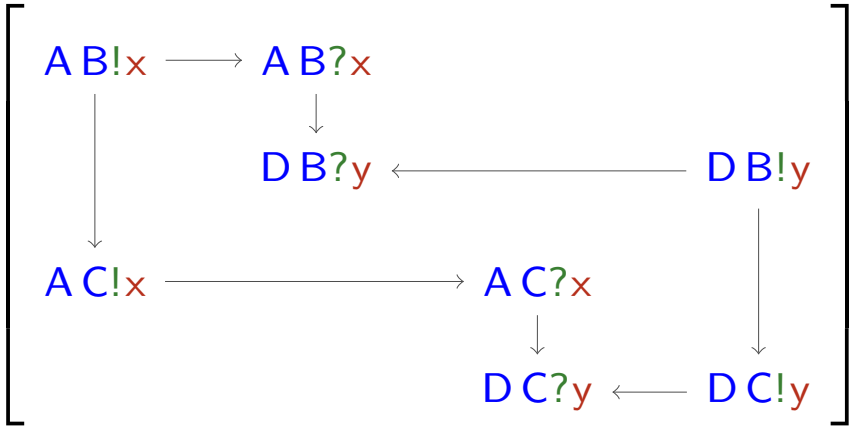



## Exercise

Find a g-choreography that corresponds to the pomset on the left.

Another unfair question!

# Pomsets semantics of g-choreographies

We define $[\![\ ]\!] : \mathsf{G} \mapsto$ set of pomsets as

$$[\![\circledcirc]\!] = \{\epsilon\}$$

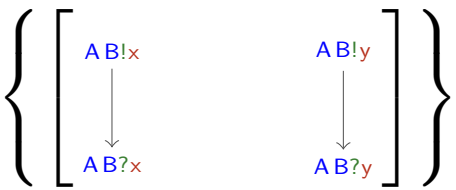
$$[\![\mathsf{A}\to\mathsf{B}:\mathsf{m}]\!] = \{[\mathsf{A}\,\mathsf{B}!\mathsf{m} \to \mathsf{A}\,\mathsf{B}?\mathsf{m}]\}$$

$$[\![\mathsf{G} \mid \mathsf{G}']\!] = \{[\text{disjoint union of } r \text{ and } r'] \mid (r, r') \in [\![\mathsf{G}]\!] \times [\![\mathsf{G}']\!]\}$$

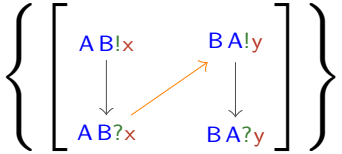
$$[\![\mathsf{G}; \mathsf{G}']\!] = \begin{cases} \bigcup_{(r,r')\in[\![\mathsf{G}]\!]\times[\![\mathsf{G}']\!]} \{\mathsf{seq}(r, r')\} & \text{if } \forall (r, r') \in [\![\mathsf{G}]\!] \times [\![\mathsf{G}']\!] : \ ws(r, r') \\ \text{undef} & \text{otherwise} \end{cases}$$

$$[\![\mathsf{G} + \mathsf{G}']\!] = \begin{cases} [\![\mathsf{G}]\!] \cup [\![\mathsf{G}']\!] & \text{if } wb(\mathsf{G}, \mathsf{G}') \\ \text{undef} & \text{otherwise} \end{cases}$$

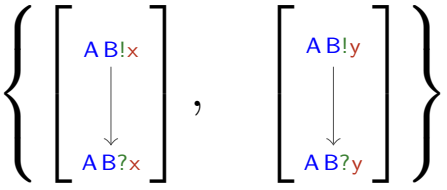
$\mathsf{A}\to\mathsf{B}:\mathsf{x} \mid \mathsf{A}\to\mathsf{B}:\mathsf{y}$




$\mathsf{A}\to\mathsf{B}:\mathsf{x};\ \mathsf{B}\to\mathsf{A}:\mathsf{y}$




$\mathsf{A}\to\mathsf{B}:\mathsf{x} + \mathsf{A}\to\mathsf{B}:\mathsf{y}$

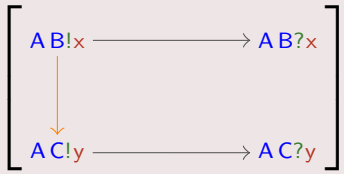


# Back to blackboard

Now we can solve the exercise on slide 18.

# Well-sequencedness

$A \to B : x; A \to C : y$



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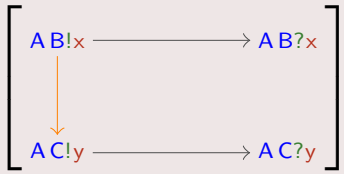
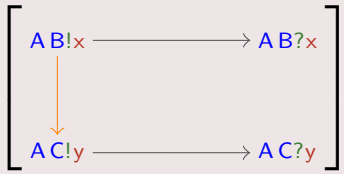



$r = [$ sbj A  sbj B $]$

$r' = [$ sbj A  sbj B $]$

# Well-sequencedness

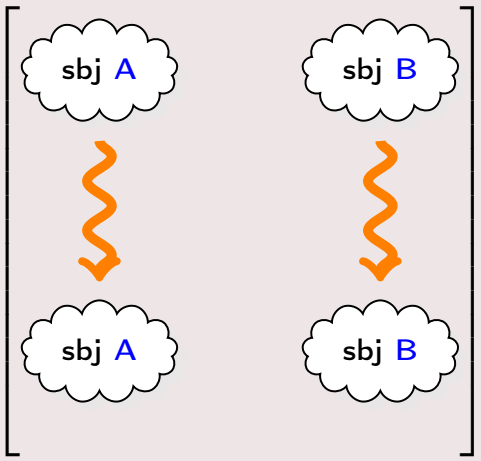
A→B : x; A→C : y




$r =$ [ sbj A  sbj B ]

$r' =$ [ sbj A  sbj B ]

$\mathsf{seq}(r, r') =$



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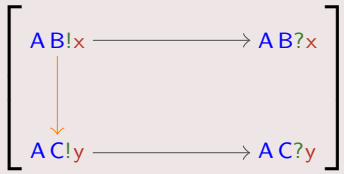
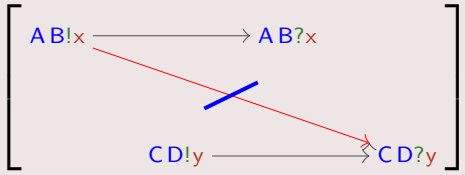



$r =$ [ sbj A  sbj B ]

$r' =$ [ sbj A  sbj B ]

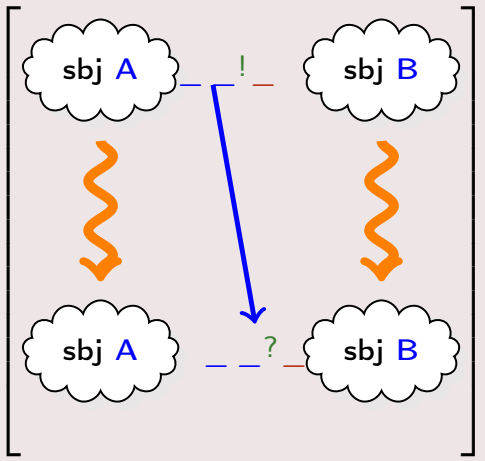
**A→B : x; C→D : y**

No minimal input "from the continuation"




(and no output in the continuation interfering with "left-inputs")

$ws(r, r')$



# Well-branchedness

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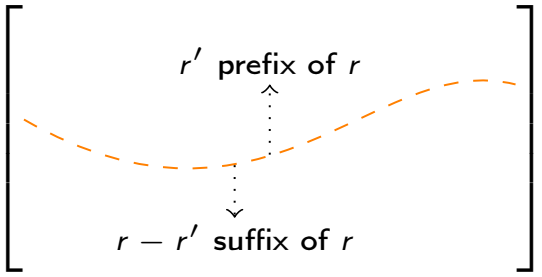
In a branch $G_1 + G_2$

- there should be at most one active participant — non-standard
- any non-active participant should be passive — standard

Problem: Participants do not necessarily "enter" a choice "immediately"
An idea: find a "common part" of the branches for which participants behaves uniformly in $G_1$ and $G_2$

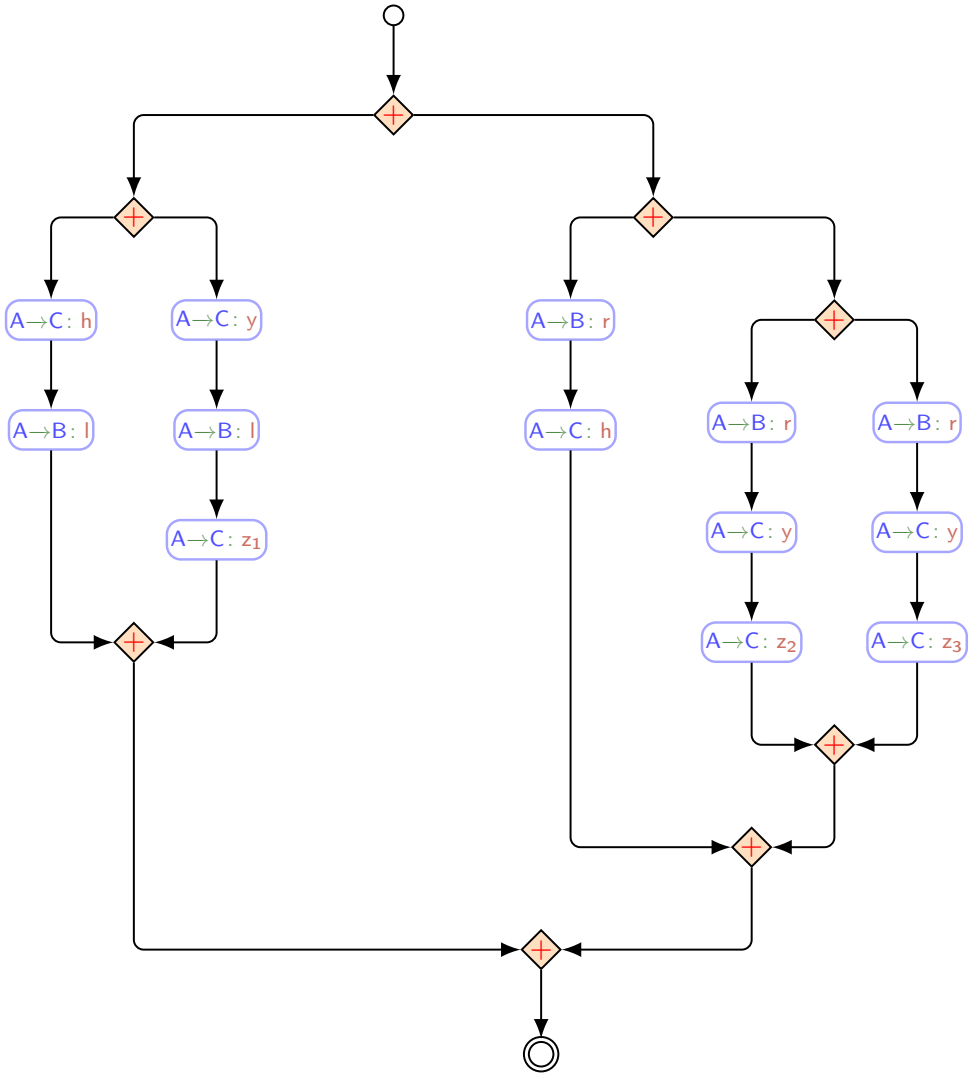
$r =$ 

# Class test

Figure out the graphical structure of the following terms and for each of them say which one is well-branched

- $G_1 = A{\to}B\colon \mathsf{int} \;+\; A{\to}B\colon \mathsf{str}$
- $G_2 = A{\to}B\colon \mathsf{int} \;+\; \circledcirc$
- $G_3 = A{\to}B\colon \mathsf{int} \;+\; A{\to}C\colon \mathsf{str}$
- $G_4 = \left( \begin{array}{l} A{\to}C\colon \mathsf{int};\, A{\to}B\colon \mathsf{bool} \\ \quad + \\ A{\to}C\colon \mathsf{str};\, A{\to}C\colon \mathsf{bool};\, A{\to}B\colon \mathsf{bool} \end{array} \right)$

# $G_{\text{😕}}$: a difficult choice




## $G_{\text{😕}} = G_1 + G_2$ where

$$G_1 = \left( \begin{array}{l} A \to C: h; A \to B: l \\ + \\ A \to C: y; A \to B: l; A \to C: z_1 \end{array} \right)$$

$$G_2 = A \to B: r; A \to C: h + G_{2a} + G_{2b}$$

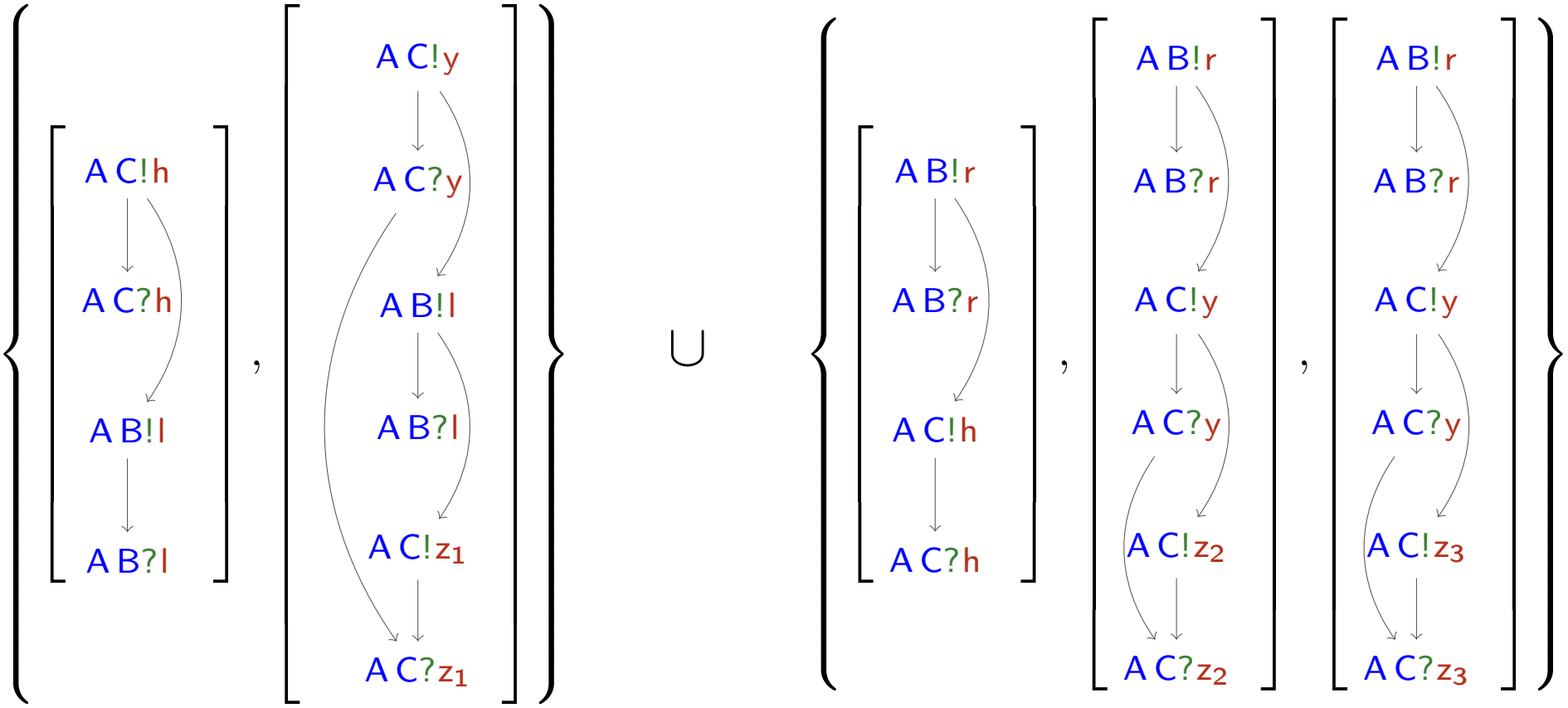
$$G_{2a} = A \to B: r; A \to C: y; A \to C: z_2$$

$$G_{2b} = A \to B: r; A \to C: y; A \to C: z_2$$

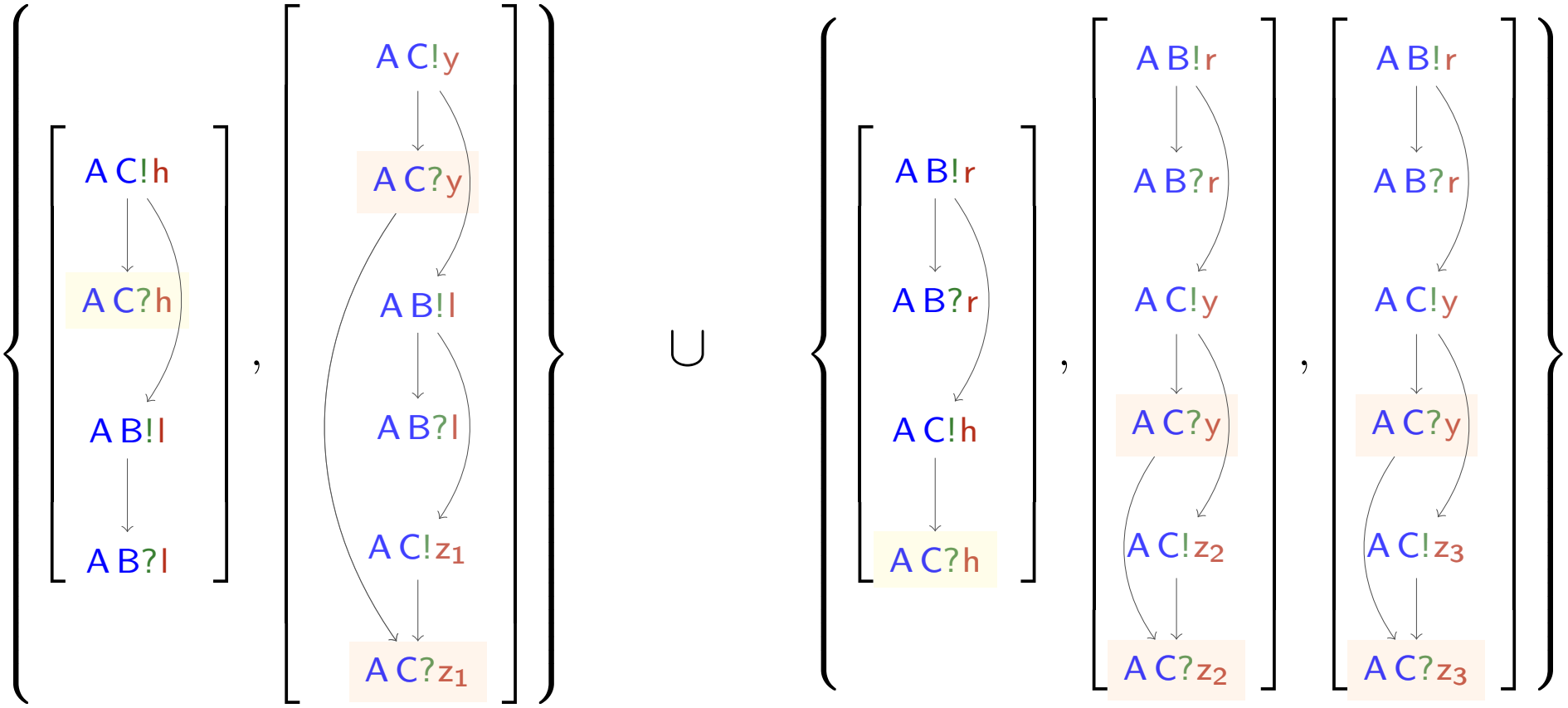
## A chooses ... and

- Whatever B gets, he won't know if A and C exchanged or not h
- If C gets h, he won't know if A and B exchanged l or r

# The pomsets of $G_{\text{😕}}$



# The pomsets of $G_{\text{😕}}$...from C's point of view



# Projecting g-choreographies

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## Technicalities

- Functions $\_ \downarrow_{\mathsf{A}}$ yield the projection of g-choreographies on the participant A as triplets $(M, q_0, q_e)$ with $q_0$ and $q_e$ initial and terminal states respectively
- If $\mathsf{G}_1$ and $\mathsf{G}_2$ are sub-terms of G then we "disjointly combine" the states of $\mathsf{G}_1 \downarrow_{\mathsf{A}}$ and $\mathsf{G}_2 \downarrow_{\mathsf{A}}$; for this we define $(M, q_0, q_e) \otimes \mathbf{1}$ which transforms each state $q$ of $M$ in $(q, 1)$ (and likewise for $(M, q_0, q_e) \otimes \mathbf{2}$)

## Base cases

$$\mathsf{G} \downarrow_{\mathsf{A}} = \begin{cases} \rightarrow (q_0) \rightarrow & \text{if } \mathsf{G} = \circledcirc \text{ or } \mathsf{G} = \mathsf{B} \rightarrow \mathsf{C} : \mathsf{m} \\ \rightarrow (q_0) \xrightarrow{\mathsf{A}\,\mathsf{B}!\mathsf{m}} (q_e) \rightarrow & \text{if } \mathsf{G} = \mathsf{A} \rightarrow \mathsf{B} : \mathsf{m}, \text{ with } q_0 \neq q_e \\ \rightarrow (q_0) \xrightarrow{\mathsf{B}\,\mathsf{A}?\mathsf{m}} (q_e) \rightarrow & \text{if } \mathsf{G} = \mathsf{B} \rightarrow \mathsf{A} : \mathsf{m}, \text{ with } q_0 \neq q_e \end{cases}$$

# Projecting g-choreographies

## Technicalities

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## Sequential composition

$$(\mathsf{G}_1 ; \mathsf{G}_2) \downarrow_{\mathsf{A}} = \left( M_1 \sqcup \left\{ {}^{q_e^1}/_{q_0^2} \right\} M_2, q_0^1, q_e^2 \right)$$

where $(M_1, q_0^1, q_e^1) = \mathsf{G}_1 \downarrow_{\mathsf{A}} \otimes \mathbf{1}$
and $(M_2, q_0^2, q_e^2) = \mathsf{G}_2 \downarrow_{\mathsf{A}} \otimes \mathbf{2}$

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## Choice

$$(\mathsf{G}_1 + \mathsf{G}_2) \downarrow_{\mathsf{A}} = \left( \left\{ {}^{q_e^2}/_{q_e^1} \right\} M_1 \sqcup \left\{ {}^{q_0^1}/_{q_0^2} \right\} M_2, q_0^1, q_e^2 \right)$$

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## Technicalities

- Functions $\_ \downarrow_{\mathsf{A}}$ yield the projection of g-choreographies on the participant $\mathsf{A}$ as triplets $(M, q_0, q_e)$ with $q_0$ and $q_e$ initial and terminal states respectively
- If $\mathsf{G}_1$ and $\mathsf{G}_2$ are sub-terms of $\mathsf{G}$ then we "disjointly combine" the states of $\mathsf{G}_1 \downarrow_{\mathsf{A}}$ and $\mathsf{G}_2 \downarrow_{\mathsf{A}}$; for this we define $(M, q_0, q_e) \otimes \mathbf{1}$ which transforms each state $q$ of $M$ in $(q, 1)$ (and likewise for $(M, q_0, q_e) \otimes \mathbf{2}$)

## Parallel composition

$$(\mathsf{G}_1 \mid \mathsf{G}_2) \downarrow_{\mathsf{A}} = (M_1 \times M_2, (q_0^1, q_0^2), (q_e^1, q_e^2))$$

where $(M_1, q_0^1, q_e^1) = \mathsf{G}_1 \downarrow_{\mathsf{A}} \otimes \mathbf{1}$
and $(M_2, q_0^2, q_e^2) = \mathsf{G}_2 \downarrow_{\mathsf{A}} \otimes \mathbf{2}$

## Exercise

Verify that the projection of the g-choreography on 4 are the CFSM on the same slide.

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